A Social Equity Lens on Toronto Transit Network Performance Using a Graph Theory Approach: Examining Criticality, Service Redundancy, Transit Delays and Disruptions

by

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Abstract

Public transit delays and disruptions are inevitable occurrences in many transit systems. Past studies did not differentiate between different groups of riders when studying the disruption impacts and rarely integrated service frequency into their network models. This study adds a social equity layer by determining if different equity-seeking groups were more vulnerable to disruptions or had less resiliency than the general population.

Various graph theory measures were used in the analysis, and both a time-expanded and an L-Space or route-map representations of the Toronto transit network were adopted. Census and travel demand survey data were used to determine trip patterns for the equity analysis.

The results show that equity-seeking riders had slightly greater vulnerability and less redundancy in hypothetical scenarios of service disruptions compared to the general population. However, when analyzing real-world disruptions that occurred in 2019-2020, equity seeking riders were more resilient compared to the general population.

Dedicated to 爸爸妈妈

Acknowledgments

The past 2 years have easily been the hardest and weirdest years of my life. When I walked out of RS310 on March 13th, 2020, I never thought that moment would be the last time I would take an in-person class at UofT, nor had I thought of the immense struggle that came during the next 2 and a half years. These people are the reason why I made it through to this point.

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Chapter 1

Introduction

1.1 Background

Congestion in major cities, climate change, air pollution, and changing urban land uses have caused cities to re-examine the public transit service they provide, as cities are increasingly reorienting their transportation strategies away from supporting private automobiles and more towards transit improvements. Many cities are opting to improve service frequency and redesigning their bus networks [88]. Federal and provincial governments are also increasing capital investment into transit [12] to introduce new transit lines, make station improvements, support fleet expansion, and promote technology upgrades. While new lines and additional frequency are important, reducing travel time delays and service shutdowns is still is a key requirement of successful public transit systems [51], as passengers need to reach their destination in a consistent and predictable manner. Otherwise, riders might perceive their destinations to be inaccessible by transit due to the risk of potential delays [24, 67], even if the scheduled travel time is reasonable, reducing participation in activities and opportunities by many who rely on public transit. Transit agencies regularly monitor and attempt to address routine (i.e., recurrent), but relatively minor service reliability issues that cause delays or travel time uncertainty. Service disruptions, such as emergency shutdowns and temporary loss in service, are equally important to consider. While they are much less frequent than minor routine delays, service shutdowns inflict significantly greater service delays, leading to high levels of rider frustration and erosion of trust in transit agencies and in public transit at large [14].

In the research area focused on transit disruptions, two gaps emerge. Many studies relating to transit disruption, network redundancy, and station criticality/importance to the network do not incorporate bus or streetcars into their analysis. Rail vehicles and lines provide more capacity to a network than surface transit lines, but in most large metropolitan areas in North America, buses serve the majority of daily ridership. Past research on transit disruptions has also been sparse on social equity. Transportation equity and justice is receiving increasingly more attention around the world in an attempt to ensure that all members of society enjoy equitable access to key opportunities and services, such as jobs, healthcare, education, or leisure [1, 9]. Considering transportation equity in public transit planning and management allows those that rely on transit as their primary mobility option or live in areas with poor transit service to have the same opportunities as those that have greater commuting flexibility and options and live in areas with better service. While many studies have focused on estimating the travel time and access to jobs for equity seeking groups, research efforts applying a social equity lens to transit reliability and disruption are rare [56, 67].

1.2 Research Objectives

This project will apply an equity lens to transit disruption effects to investigate whether equity seeking groups are more vulnerable to service disruption, and whether they face limited network redundancy or path choices during disruption relative to the General Population. The scope of this project includes all modes of a multimodal transit network, recognizing that many equity seeking riders rely more on buses than subways or streetcars [79], and are therefore more affected by delays and disruptions on bus routes.

Many tools and measures used in past disruption research assume a non-dynamic transit schedule and ignore the impact of headways. Those assumptions are less appropriate when analyzing a multimodal network, since some bus and streetcar routes have large variability in service headways at different periods of the day, and many bus routes have low frequency services during off peak periods when members of some equity seeking groups use transit for essential mobility needs. This study attempts to address this limitation and it also adapts existing graph theory measures of network robustness and resilience to support our analysis.

1.3 Thesis Structure

This thesis is divided into seven chapters. Chapter 2 presents a literature review on three main topics:

1. Concepts of transit disruption and synthesis of previous research

2. Graph theory approaches to representing transit networks and graph theory measures of network characteristics

3. Transportation equity as it relates to public transit

Chapter 3 delves into the methodology of the thesis, including the structure of Toronto's transit network, how the various graphs used in the project are built, what data is used and how the data is transformed, and what graph theory measures are used in the project. Chapters 4, 5, & 6 present the results of three different sub-analyses. Chapter 4 analyzes how vulnerable equity seeking groups are to disruptions by investigating how those groups are more concentrated onto specific stations or routes. Chapter 5 explores the level of network redundancy and path options experienced by equity seeking groups relative to the General Population. Chapter 7 applies some of the methods used in Chapter 4 using real time data from the winter of 2019 instead of GTFS feeds. Finally, Chapter 7 concludes the thesis by summarizing the results, discussing policy implications, and highlighting the limitations of the thesis.

Chapter 2

Literature Review

2.1 Introduction

The literature review is divided into three sections. The first section will discuss concepts related to public transit disruptions. In doing so, it demonstrates how previous authors used graph theory for their disruption analysis. The second section introduces graph theory, and common measures and indicators used in graph analyses. The last section introduces the need for transportation equity, and what types of methods past literature used for transportation equity studies.

2.2 Disruption

2.2.1 Disruption Concepts

In a public transit system, there are many operational factors that affect a passenger's experience. For example, speed is one factor that impacts the travel time a passenger takes when travelling from their origin to their destination. Headway affects how much a passenger waits for their bus or train when making a transfer or their first leg of their journey. While speed and headway are important, this section will focus on other important factors, specifically the concept of reliability and the related concept of disruption.

Reliability can be thought of as the variation of transit service across different states of time [86]. Common ways to measure reliability is the on-time performance of a route, or the length of delay a route accumulates. For transit riders, reliability is one of the most important factors in the attractiveness of public transit as a mode [44]. Being aboard a late bus or train can make a passenger late for their event or appointment, and research has shown that passengers will increase their wait and transfer time to account for unreliable service [57]. This effectively adds extra time to their travel time, making transit a less attractive option compared to other modes that are faster and more reliable, such as driving or ridesharing.

A related concept to reliability is the concept of disruption. Disruption is similar to reliability in that both concepts are about variation in serviceability over a period of time; unreliable service describes recurring perturbations in service while transit disruptions describe sudden service changes and shutdowns due to non-recurrent events. Disruptions tend to cause more devastating impacts to the transit network and require a longer timeframe for the network to recover back to its normal state [8, 61, 80].

Unreliable service is typically caused by cascading delays from arriving too late or too early at a station or stop [60]. Two common causes of this can be traffic, and variability in dwell time, which are events that happen daily and on a regular basis. Common events that cause disruption are targeted attacks or sabotage, terrorism, extreme weather, infrastructure failure, unique travel demands such as sporting events, and road/track closures [61, 46]. For rail networks, additional causes can be operator or dispatcher error, track injuries, door failures, trespassing, track or train failure, fire, passenger injuries, and other security threats [91, 66]. These causes have shown that the causes of disruptions can be both anthropogenic and environmental and can be either targeted or random. Similar to reliability, these events have negative experience on riders. However instead of adding minutes to their travel time, disruptions can easily add hours.

Within disruption research, researchers have focused on various concepts that describe various elements of disruptions. Vulnerability is defined as the susceptibility of a station or line to disruption [61, 46, 91]. For example, newer segments of subway lines typically experience less disruption than older segments, thus having a lower vulnerability. A heavily trafficked road has a high vulnerability to disruption since the probability of a collision is higher. In the winter, the entire network is more vulnerable due to the possibility of snow and ice causing breakdowns in bus routes or rail services. Importance, otherwise known as criticality, is how important a station is to the transit network [91], and how reliant passengers are on that specific station or route segment. Disruptions affecting critical stations will cause a larger negative impact on the network since it will disrupt more passengers than a less critical station.

Resiliency is the speed at which the network returns to its normal state [61, 26] and can be thought of as the integral of time and capacity [80]. For example, a disruption that forces a year long station closure will have a lower resiliency than a disruption that causes a four hour long line closure, depending on the ridership and capacity of both the line and station. Higher resiliency usually means the system is able to absorb shocks and reductions in capacity and is related to the redundancy of the system.

Redundancy, otherwise known as robustness, is excess capacity in the system that is not normally used in normal conditions [46]. This can manifest in extra capacity aboard buses and trains, or multiple routes serving the same area. Excess capacity becomes important during disruption. Passengers can choose alternative routes, provided they have capacity to absorb the extra passenger demand, to travel to their final destination if their primary route is disrupted. This can significantly mitigate the length of their delay if redundant service is provided to passengers.

2.2.2 Examples of Disruption Research

Many studies have addressed transit disruptions, and their methods and limitations can be addressed in creating the methods for this thesis.

Rommel [45] investigated the topological properties of the Stockholm metro network. The analysis was conducted from 1950, the date the first line opened to 2025 when the fourth metro line will be completed. The authors used the graph theory concepts of betweenness centrality, closeness centrality, degree centrality to investigate the importance and vulnerability of the Stockholm metro. They also computed the average nearest node degree, the clustering coefficient, the pearson coefficient, and the connectivity to determine how robust the network is. They found that as more expansions were added to the metro network, many individual stations became less important, the redundancy of the system was increased, and the network would have an increased ability to resist disruptions. However, the thesis did not incorporate ridership of each station/line or the number of OD trips into the analysis. Incorporating ridership could make certain stations more or less critical than what their betweenness centrality results show.

Serlier [72] conducted a thorough analysis on the route-segments of the London Underground. The author removed route-segments from the rail network, then determined the travel time delay faced by passengers having to use an alternate route. The study found that the crossings of the Thames River were the most important sections of the London Underground and would cause the most disruption if affected. A similar exercise was conducted by King [50] in Toronto. The author used a travel demand model built in simulation software EMME for the analysis. The author found that a disruption at Lawrence station, which did not have the highest ridership in the Toronto Subway system, would cause the most delay to passenger trip pattern due to the number of passengers relying on suburban bus routes and the lack of alternative north-south bus routes or subway lines. The author also used a graph model to evaluate global efficiency, an indicator that measures how well riders can get to their destination. After randomly removing 15% of edges in the network, a 50% loss in global efficiency would occur. However, global efficiency did not take into account the number of passengers on each origin-destination pair. Sun et al. [77] also conducted a global efficiency analysis of the Shanghai Metro. They reached a similar conclusion to King [50], in that the stations with the highest ridership are not the most important to the network. Like Rommel [45], these three studies only analyzed the metro network.

Derrible and Kennedy [33] compared the connectivity, directness, and coverage of 19 metro networks around the world. They found that more redundancy and more connected metro networks usually correlate to higher per-capita boardings. In a similar paper, Derrible and Kennedy [32], compared the connectivity of the future 2035 Toronto Subway and LRT network to the 2025 network, the 2010 network, and the 18 other metro networks around the world studied in their earlier paper. They were able to predict the ridership the future Toronto rail network would have based on the connectivity score and the level of redundancy calculated for the 2010, 2025, and 2035 networks. However, these two studies focused only on the metro and LRT network.

Cats and Jenelius [13] attempted to incorporate the number of riders using a particular station or route in their importance analysis. They created two new measures, passenger betweenness centrality and operator betweenness centrality. Passenger betweenness centrality identifies the road segment with the highest passenger throughput among all routes, while operator betweenness centrality identifies the road segment with the highest frequency level among all routes. A similar analysis was done for the Shanghai Metro Network by Sun & Guan [78]. In both studies, the authors used a traffic assignment model to determine ridership on each route segment to find which segment was the most critical to the transit network.

2.2.3 Gap in Literature

Many disruption studies have focused on the rail network, despite the significance of non-rail transit ridership. Many disruption studies have also failed to consider how trip flows can change the vulnerability and redundancy analysis that they present. The two gaps demonstrate a need for a study that incorporates OD trip flows and analyzes a multi-modal system with non-rail transit modes.

2.3 Graph Theory

2.3.1 Graph Theory Introduction

As seen in Chapter 2.2, many researchers have used graph theory for their disruption analysis. This subsection provides a background on graph theory. Graph theory and topology was first theorized by Euler in addressing the "Königsberg Bridge Problem" [2]. The premise of the problem was to develop an itinerary such that a parade crosses each of the seven bridges in Königsberg only once. Euler represented the problem as a network. Each of the bridges in the network was represented as a series of edges or links, and the island and the two sides of the riverbank were represented as nodes.

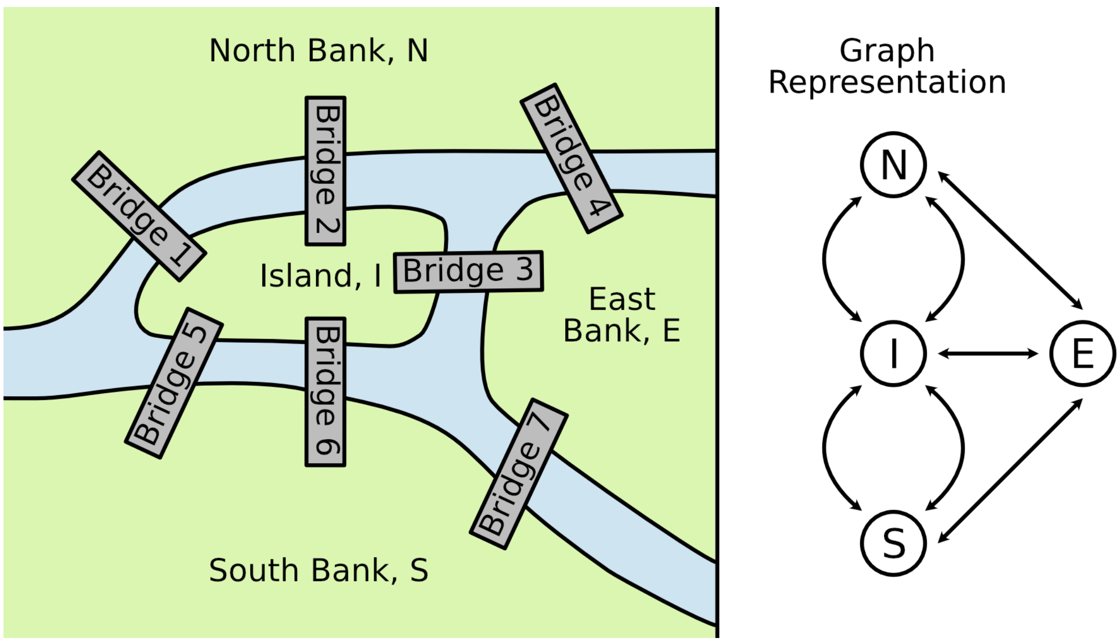

Figure 2.1: Map and graph representation of the seven bridges of Königsberg.

This spawned the field of topology where a series of measures or algorithms can be applied to a network made up of nodes and edges to solve routing or network problems. Garrison and Marble [38] applied topology to a transportation context and developed new measures specific to highway networks. Lam and Schuler [52] along with Musso and Vuchic [63] applied the techniques developed by Garrison and Marble to transit networks [31], while also creating transit specific measures for pathfinding and transit network design.

2.3.2 Types of Graphs

While there are many implementations of graph theory in the transit context, authors have developed different ways to represent transit networks by choosing different graph properties, or types of graphs. Most authors choose to represent their transit network in an L-space graph. In an L-space, each station or bus stop is represented by a single node, and the routes connecting each station or stop are represented by links or edges [58, 36, 8, 80]. Edges are connected sequentially; stations served by only a single route would only be connected to the preceding station and succeeding station on the route. This graph type represents the network like a system map of a transit agency.

In a P-Space graph, nodes also represent stations or stops, but they are connected to all other nodes reachable on the route serving that station or stop [36, 41, 8, 80]. In a B-Space graph, nodes represent stations, stops and routes, while edges represent the interaction between routes and stops. If a route serves a stop, the route node is connected to the stop node [36, 45, 31]. Other types of graphs exist; however, they are less common in the public transit context.

Graphs and graph edges can be directed or undirected. If a directed edge connects an origin to a destination, a path can be drawn from the origin to the destination but not the destination to the origin [53]. Graphs can also be planar and non-planar. In a planar graph, edges cannot cross over one another, while in a non-planar graph, edges can overlap and cross other edges without a node connecting the two edges. Most subway or rail networks can be represented as a planar graph. If a graph represents both a bus and subway network, then the graph would need to be non-planar since many cities have bus routes that cross but do not connect to other subway lines or bus routes.

2.3.3 Graph Theory Measures

Different authors have come up with a number of different measures computed from graph networks in order to describe the overall state of the network.

Structural connectivity is a concept that describes how connected the network is to other nodes [45, 33, 42]. In the transit context, connectivity can be computed as the sum of all transfer opportunities; a station or stop served by more than one route has at least one transfer opportunity. The sum of transfers can describe how easily a transit user can travel throughout the system by transferring to other routes. Equation 2.1 and 2.2 shows the formulation for connectivity.

$$v_{tr} = \Sigma_i[(d_i - 2)\frac{d_i}{2}] \tag{2.1}$$

Where

- v_{tr} = transfer possibilities
- d_i = degree of node i

$$\rho = \frac{v_{tr} - e_m}{v_t} \tag{2.2}$$

Where

- ρ = connectivity
- v_{tr} = transfer possibilities
- e_m = number of edges with more than one route
- v_t = number of nodes served by more than one route (transfer nodes)

Clustering measures how well connected the neighbours of a node are to other neighbours of the node. It can be calculated by counting the number of triangles the node of interest forms with its neighbours [71, 36, 8, 42, 6]. A similar measure is the average neighbouring node degree. This can be calculated by taking the average degree, defined as the number of edges connected to that node, of all nodes neighbouring the node of interest [6]. Both metrics aim to measure if there is alternative transit service easily reachable from a specific node. Equation 2.3 shows the definition of clustering, while Equation 2.4 shows the definition of average neighbouring node degree.

$$C_i = \frac{2T_i}{d_i(d_i - 1)} \tag{2.3}$$

Where

- C_i = clustering coefficient of node i
- T_i = number of edges connecting nodes that are neighbouring node i, or triangles
- d_i = degree of node i

$$\overline{nnd_i} = \frac{\sum_j d_j}{d_i} \tag{2.4}$$

Where

- $\overline{nnd_i}$ = average degree of the nodes neighbouring node i
- d_i = degree of node i
- d_j = degree of node j
- j = neighbouring nodes of node i

Many authors have used centrality measures such as eigenvector, degree, or closeness centralities [42, 62]. However, the most common measure is betweenness centrality (BC). For a given node, BC measures the proportion of shortest paths between origin and destination (OD) pairs that pass through the node [58, 45]. This measure aims to find how critical the node is to transit users travelling through a network; a node with a high BC will usually have a higher throughput of riders, with some exceptions. The equation of BC is shown in Equation 2.5.

$$BC(i) = \frac{\sum_{od} \frac{n_{od}(i)}{n_{od}}}{(N-2)(N-1)} \tag{2.5}$$

Where

- $BC(i)$ = betweenness centrality for node i
- n_{od} = number of shortest paths between an OD pair
- $n_{od}(i)$ = number of shortest paths between an OD pair crossing intersection or station i
- N = number of intersections and stations in the graph
- od = origin-destination pair

Global Efficiency (GE) sums the inverse path length, defined as travel time in the transit context, for a specific OD pair, which is then summed over all OD pairs [50, 54]. Generally, the measure describes how easily passengers can reach their destination. As total travel time increases, the corresponding GE would get smaller. Compared to summing total travel time or path length, GE allows for cases where the travel time or path length cannot be calculated because the travel time is too long; this case would produce a GE of zero. Global efficiency is defined in Equation 2.6.

$$GE = \sum_{od} \frac{1}{c_{od}} \tag{2.6}$$

Where

- GE = global efficiency

- c_od = shortest path travel time for a specific origin destination pair
- od = origin-destination pair

Some authors have taken into consideration the number of paths an OD pair has. Jing et al. [47] considered the fact that many paths involving transit overlap with one another, and they described a process to determine the effective number of paths after considering route overlapping. Equation 2.7 and 2.8 shows the algorithm to compute the effective number of paths.

$$\gamma^{od} = \begin{cases} 1 & p^{od} = 1 \\ 1 - \frac{\sum_i \ln p_i^{od} * c_i}{\sum_i \ln p^{od} * c_i} & p^{od} \neq 1 \end{cases} \tag{2.7}$$

Where

- γ_{od} = overlapping degree for an OD pair
- p^{od} = raw number of paths for an OD pair
- p_i^{od} = raw number of paths for an OD pair passing through edge i
- c_i = cost of edge i
- i = edge

$$ep^{od} = 1 + \gamma^{od} * (p^{od} - 1) \tag{2.8}$$

Where

- ep^{od} = effective number of paths between an origin and destination

2.3.4 Incorporating the Time Dimension into Graphs

Many studies involving graph theory and transit have analyzed rail networks or metro networks [31]; however only a few have analyzed bus networks. Hong et al. [42] studied the Seoul bus and metro network as an integrated system and represented it as an L-Space graph. Zhang [92] analyzed bus networks in four cities in China, Baoding, Jinan, Shijiazhuang, and Suzhou, by creating a P-Space graph. Shanmukhappa et al. [73] created an L-Space representation of bus networks in London, Hong Kong, and Bengaluru. The author also created "supernodes" to simplify journeys involving transfers. Bus stops close to other bus stops, such as at four corners of an intersection, were represented as a single node instead of four separate nodes.

In all of those studies, the issue of frequency and headways was ignored. An edge connecting two nodes has an implicit assumption that travel is possible at all times of the day, since graphs do not normally carry a time dimension. That assumption can be justified for most metro networks. Metro trains arrive and depart in a frequent enough manner that most passengers do not need to time their transfers or departure times. However, many bus networks have a large number of low-frequency routes that operate at headways of 30 minutes or longer. In those cases, it is inappropriate to suggest that a connection always exists between two nodes. In addition, frequency can vary greatly depending on the time of day, since many routes draw down their service during off-peak hours. In order to consider the time-of-day variation of transit service, Maduako et al. [59] developed 165 graph snapshots to represent a single day of transit in Moncton. Each snapshot contains a graph that represents a different state of time and was created using automatic vehicle location (AVL) data from the GPS coordinates of each vehicle. This addressed the issue of variations in service depending on the time of day.

Quintero-Cano [69] attempted to incorporate the frequency of Vancouver transit routes by weighing the edges by frequency. The study represented the graph as an L-Space graph, with nodes representing bus and train stops and edges representing connections between nodes. Each edge was weighted by the frequency of all routes along that edge over the most frequent edge in the TransLink network. Similar to Shanmukhappa [73], bus stops and train stations were consolidated to a supernode if stops were located near each other to simplify the representations transfers. Graph measures, such as connectivity and complexity, were used to produce a model to predict the number of collisions.

Another approach to represent headways is to add a time dimension to the graph, creating a Time-Expanded graph [7]. In the transit context, each node represents a stop or station at a specific instance in time, while an edge represents a connection available at that time. Compared to weighting each edge with a frequency factor, a time expanded graph is more accurate for calculating paths and path lengths or travel times.

Fortin et al. [16] applied a time-expanded graph to a small transit network in Chambly, Quebec. Each stop was represented by a node, and edges were created to connect nearby nodes at the same intersection to facilitate transfers.

Whited [10] applied a time expanded graph to the larger Edmonton Transit Service. The author also classified L-Space graphs as either a route-map graph, if the edges represented individual routes, and trip-map graphs, if edges represented individual vehicles or trips and multiple edges exist between the same origin and destination.

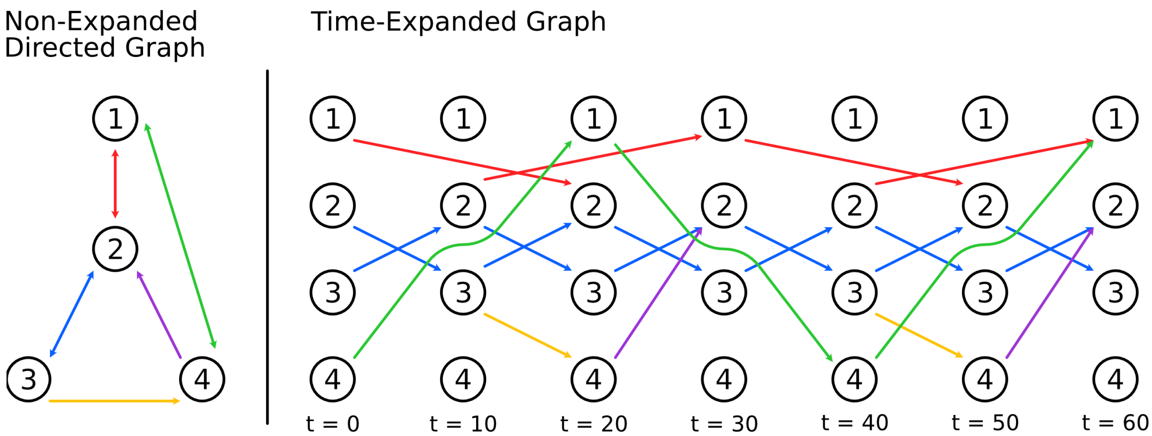

Figure 2.2: An L-Space graph (left) alongside a time expanded graph (right).

Like Fortin [16], Whited [10] drew edges connecting stops located at the same intersection or transit hub to consider transfers between two routes. Whited did not calculate betweenness centrality for a time-expanded graph; a time-expanded graph would not be compatible with many graph measures as traditionally defined, since an individual node does not represent a stop or station, but a stop at a state in time.

2.3.5 Gap in Literature

Applications of graphs that consider headway remain sparse, and studies that have created a graph that incorporated headways and frequency have not reached the same level of detail or use the same techniques and measures as traditional studies that focus on an L-space graph. While time-expanded graphs are not new in, their use in transportation network literature remains relatively primitive.

2.4 Transportation Equity

2.4.1 Transportation Equity Definitions

In the discussion surrounding transportation equity, equity can either be classified into horizontal equity or vertical equity. Horizontal equity is described as ensuring various groups receive the same benefit [55, 9]. Effort is made to avoid discriminating among groups and ensuring the same benefits are received equally [39]. In the transit context, this may manifest as ensuring that transit travel times between High-Income users and Low-Income users are equal or ensuring that both groups have the same level of access to a station. A better term for horizontal equity may be equality, as it relates to the equal distribution of a good or service. On the other hand, vertical equity ensures that a fair distribution of service occurs [9, 39] and limits the regressive distribution of negative impacts onto disadvantaged groups [55]. In the above example, an equitable transit system would aim to have better travel time, and improved access to transit stations for lower income individuals, since it recognizes that transportation costs make up a greater burden for Low-Income individuals, and Low-Income users usually do not have access to other modes such as driving. For this thesis, vertical equity shall be referred to as equity, while horizontal equity shall be referred to as equality. Vertical equity is more aligned with the strictest definition of equity: a system where individuals and groups are treated fairly as opposed to equally [9].

2.4.2 Justification for Equity

The need for equity in public transit has been made clear in prior literature. Transit can provide access to opportunities and amenities [55], such as essential services such as health or groceries, greenspace, social interaction, or jobs [1, 43]. These opportunities can allow residents to participate according to their full potential in society [9], allowing for individuals to be more included in society [48]. Liberalistic societies value a right to life, which implies the right for individuals to be included in society, while egalitarian societies may aspire to have each individual access the same opportunities no matter their background [9]. These two differing lenses demonstrate two different rationales for the role for transportation to be provided fairly and equitably to ensure fair treatment and universal participation in society. Survey data has shown that many Low-Income individuals turn down jobs, due to the length of their commutes, and Low-Income individuals face barriers towards relocating to reduce their commute due to their limited assets and wealth [24]. Equity would ensure that better transit is provided to Low-Income individuals so they could access the same level of opportunities and participation in society as more affluent individuals.

Past decision making in transit has not been equitable and has been regressive towards those that need transit the most. Low-Income riders often make up the lion's share of riders, however transit improvements tend to focus on projects that would benefit affluent riders, or those that have the choice between transit and private automobiles [79]. This is usually seen in transit prioritizing rail projects over improved bus routes, or prioritizing commuter rail in affluent suburbs over the Inner City and Low-Income neighbourhoods. The focus on attracting affluent riders hurts transit reliant users, since they do not have alternative options and have a far greater need for transit than those that have the ability to choose between driving, walking or cycling.

2.4.3 Population Groups of Interest for Equity Analyses

For equity analysis in transit, there also is a discussion on who is an equity seeking user, and who has the greatest need for transit. The Transit Cooperative Research Program has published a guideline on the steps for agencies to perform an equity analysis in accordance with Title VI of the Civil Rights Act of 1964. They describe the groups of interest as minorities (also described as Racialized individuals or Visible Minorities), Low-Income groups, and those with a limited English proficiency [43]. Some agencies have used a threshold of twice the federal poverty line as the definition for Low-Income individuals for their Title VI analysis. Other researchers have added Youths, Seniors, Recent Immigrants, those with a disability, Carless Households, and Women as equity seeking populations for a transit equity analysis [51, 9, 55] in addition to the existing three groups of interest.

2.4.4 Past Transit Equity Studies

Many transit equity studies focus on the accessibility and travel times to jobs or other opportunities. A common approach was to compare the travel times using an accessibility measure to low and high wage jobs, then comparing if the differences are equitable [49, 9]. This exercise has also been done in Canadian cities, such as Toronto [39, 37, 1], and other smaller cities in Canada [1]. In general, these studies found that equity seeking individuals in Canadian cities have similar accessibility or better compared to the General Population.

In contrast, the availability of transit equity literature with a focus on reliability and disruption is sparse, however a reliable transit system is still a key consideration for transit users [51, 44]. Equity seeking users often find themselves employed in precarious situations and receive harsher consequences for lateness, such as dismissal or reprimands, compared to more well-off users [24]. Unreliable transit systems, and systems that are highly vulnerable to disruptions, will cause riders to add extra buffer to their trips and accessibility literature that do not consider the impact of unreliability/disruptions can overestimate access due to the extra travel time needed [57, 67, 90], and lowers ridership on unreliable routes [15]. Since service can often vary by up to 20% on a day without transit disruption [90], it is important for equity analysis to consider the effects of disruption and unreliability. Besides studies done to determine if a transit agency's response to subway shutdowns were equitable [56], and whether disadvantaged populations were more prone to late buses than the General Population [67], social equity studies on the topic of disruption and reliability remain rare.

Other studies exist on utilizing a Lorenz curve or the Gini coefficient to determine if a network is connected and redundant, which will make a transit network more resilient in the face of disruption. This has been done for Melbourne [29], Tennessee [74] and the Washington DC-Baltimore area [89]. These studies tend to focus more on equality and horizontal equity [9], since they assessed whether equal levels of connectivity are felt among all percentiles of the population.

2.4.5 Gap in Literature

Past studies have shown a wealth of studies considering transit equity concerning travel time, and accessibility to jobs. Yet ensuring reliability and mitigating the effects of disruption are equally important considerations in public transit operations and planning. The literature review on equity has shown a need for a study that applies equity concepts to transit disruption research.

2.5 Literature Review Conclusion

After performing the literature review, multiple gaps in the research were observed.

- Few studies analyzed the redundancy and importance across all hours of the day, as opposed to the peak period.
- There were relatively few disruption studies that analyzed all modes of a transit network, such as buses, light rail, and heavy rail, due to the limitations of non-time expanded graphs.
- Few graph studies appropriately considered the impact of frequency on the results, since many graph studies focus on high frequency rail transit networks.
- Few studies exist that researched the equity impacts of disruption and reliability and how different groups of people are affected by disruption.

Combined, there exists no literature that addresses all of the above gaps in a single literature. This thesis presents an opportunity to create a project that will use a social equity lens to analyze the redundancy of a multi-modal transit network, and the importance of individual routes and stations in the network, while also incorporating frequency in the analysis. Chapter 3 will discuss how the methodology the thesis will use to address the gaps, while Chapters 4, 5, 6 will discuss the results of the analysis.

Chapter 3

Methodology

3.1 Introduction

This chapter discusses the methodology of the thesis to address the four gaps demonstrated in the literature review. The analysis was divided into multiple phases to analyze the impacts of disruption.

1. Importance and criticality of stations and routes of the 2016 Toronto transit network. The results are discussed in Chapter 4.

2. Redundancy of the 2016 Toronto transit network. The results are discussed in Chapter 5.

3. Impacts of disruption and unreliability on riders using real-time data from November 2019 to January 2020. The results are discussed in Chapter 6.

For the three phases of the project, different types of graphs were required. A route-map graph was used for phase 2, while a time-expanded graph was used for phases 1 and 2. Both graphs were based on data from the 2016 October 3rd GTFS Feed. Phase 3 of the project also required time-expanded graphs, but instead of a static graph based on a single GTFS feed, 21 time-expanded graphs were created, each representing a different weekday from November 28, 2019, to January 3rd, 2020. These graphs were not based on a GTFS feed; instead, they were based on real time train or bus arrival data pertaining to that timeframe. To serve as a baseline, phase 3 also used a time-expanded graph based on the November 24th, 2019, GTFS feed.

Phase 1 used a modified betweenness centrality measure, and a modified global efficiency measure. Both measures were modified to incorporate origin destination (OD) trip flows, which was accomplished through the use of OD survey data and census data. Phase 2 used the following measures: connectivity, average neighbouring node degree, and additional path factor. Like Phase 2, Phase 3 also used the modified global efficiency measure.

For all three phases, five analysis periods were considered:

- Early Morning (EM), with trips departing between 4:00 and 5:00
- Morning Rush Hour (AM), with trips departing between 7:00 and 8:00
- Mid-day (MD), with trips departing between 11:00 and 12:00
- Afternoon Rush Hour (PM), with rips departing between 17:00 and 18:00
- Evening (EV), with trips departing between 20:00 and 21:00

3.2 Study Area

The Toronto Transit Commission (TTC) network was taken as the network of interest. While the TTC does serve areas outside of Toronto, the project was focused only on service inside the City of Toronto. The TTC is Canada's largest transit system, and with an average weekday ridership of 1.7 million in 2019 it is the second largest transit system by ridership in Canada and the United States, after New York's MTA [3]. The TTC is a multi-modal network composed of 191 bus routes, 14 streetcar routes, and four subway lines.

Figure 3.1 contains a map of TTC transit routes in the City of Toronto, while Figure 3.2 shows the boundaries of each ward in Toronto city council and the definition of Toronto districts used in this study. These districts follow the Toronto community council boundaries, with the exception of the Toronto-East York community council, which was divided into the two districts: CBD and Inner City Toronto. Certain bus and streetcar routes, and all subway lines, are classified as belonging to the 10-minute network: a network of frequent surface transit routes with headways 10 minutes or less whenever the subway is operational. The four subway lines operate from 6:00 to 1:00; during the overnight hours, subway lines are replaced by bus routes as part of the Blue Night Network. Certain bus and streetcar routes also run 24 hours a day. Unlike most North American light rail systems, the vast majority of Toronto's legacy streetcar system runs in a shared right of way, has stop spacing similar to bus routes, and shares many operational characteristics as the bus network. Streetcar routes are concentrated in the pre-1997 boundaries of Toronto, otherwise named Old Toronto.

For the study, subway stations were classified into two categories. Stations located within Old Toronto were classified as inner subway stations, while subway stations located outside of Old Toronto (in the districts of Etobicoke-York, North York, and Scarborough) were classified as outer subway stations.

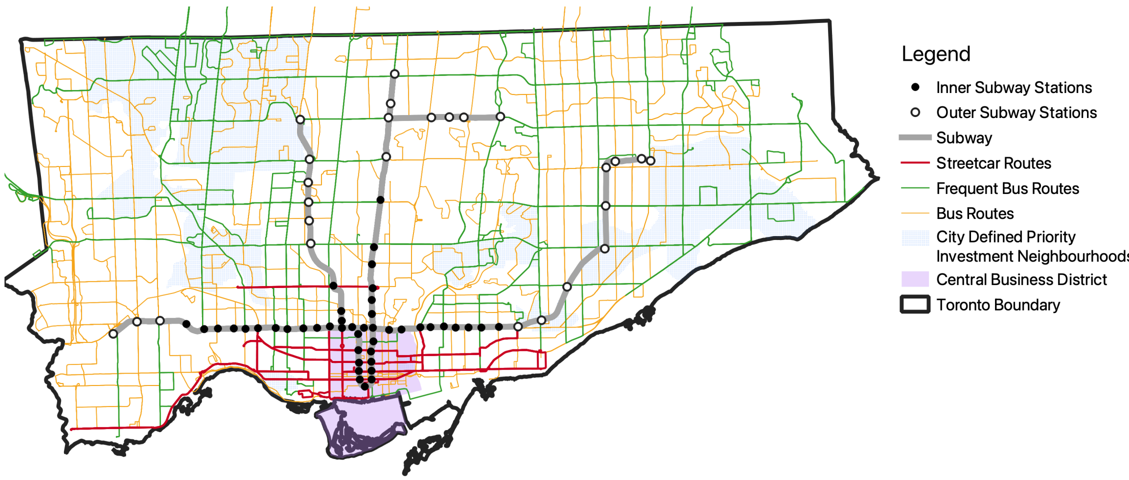

Figure 3.1: Map of the TTC network. [21, 83, 20]

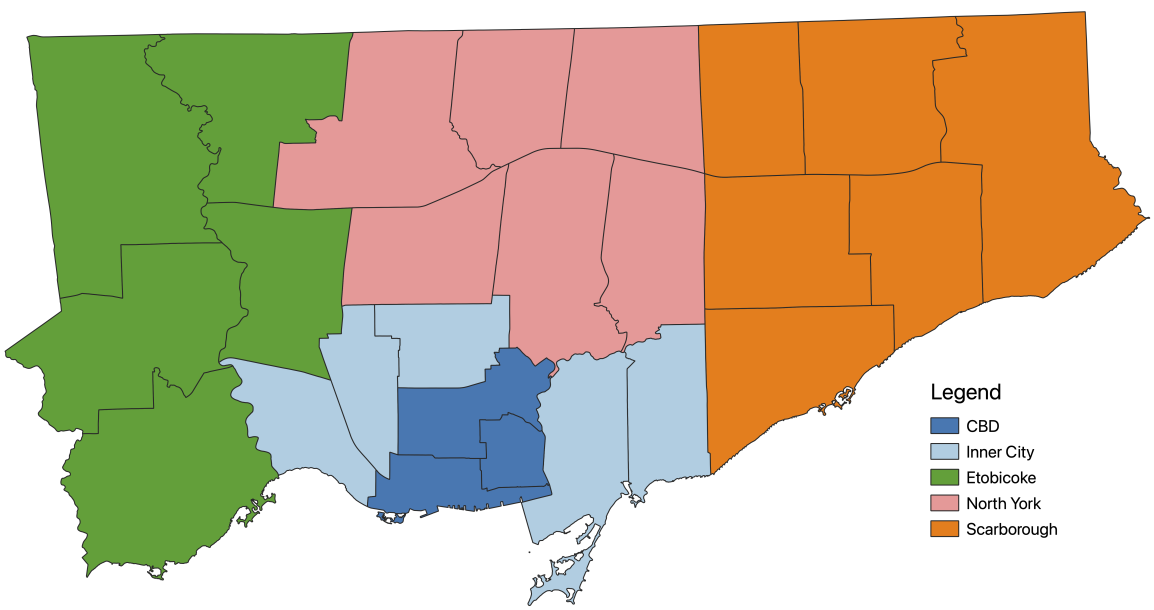

Figure 3.2: Map of Toronto districts, ward boundaries. For the purpose of the study, Old Toronto was divided into two districts: the CBD and the Inner City. [17, 18]

Figure 3.3 shows the service levels as a percentage of peak hour service levels. The figure was computed from summing in-vehicle service hours from the GTFS feed. Like most transit agencies, service peaks during the two rush hour periods. The midday period experiences a slight draw down, while service in the evenings draws down to a much lower level than the midday period. Subway and bus routes generally follow a similar service level pattern, while streetcar routes do not experience as large of a service drawdown during the midday period.

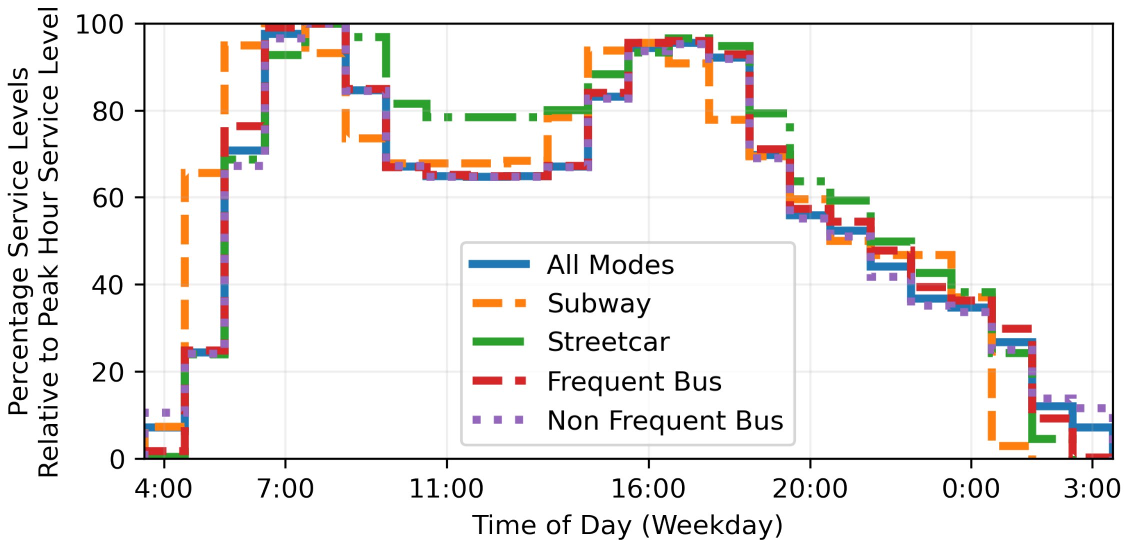

Figure 3.3: Weekday service levels of the TTC by time of day. [83]

3.3 Equity Seeking Groups Selection

Similar to the guidelines set out in TCRP Report 214 [43], the following population groups were defined as equity seeking groups:

- Low-Income
- Racialized (including Visible Minorities and Indigenous individuals)
- Limited English Proficiency or LEP (The 2016 Canadian census describes this group as "No Official Languages Proficiency" and counts those that are not proficient in either English or French. This census does not include those that have a mother tongue other than English or French but can still speak English or French at a usable level.)
- Black Canadians
- Recent Immigrants (those that had immigrated from 2011 to 2016 according to the 2016 census)
- Carless Households

Household Size	LICO Upper Bound	150% of LICO	Project Defined Upper Bound	LICO Factor
1	20,675	31,012	39,999	193%
2	25,163	37,744	39,999	159%
3	31,334	47,001	59,999	191%
4	39,092	58,638	59,999	153%
5	44,514	66,771	99,999	225%
6	49,367	74,051	99,999	203%
7+	54,220	81,330	99,999	184%

Table 3.1: Definition of Low-Income households for the project.

For the Low-Income population group, Table 3.1 defines the upper bounds of households that can be considered Low-Income for the study. Like the example analyses in TCRP 214, a definition of 150% of the Statistics Canada Low-Income Cut Off (LICO) [11] was used as the baseline definition of Low-Income households to account for the high cost of living in Toronto; however, the upper bounds were adjusted to include more households since disaggregate income data in the OD travel survey was not available. Statistics Canada LICO upper bounds were prescribed for urban areas over 500,000; as Toronto the population of Toronto is well over 500,000, a higher upper bound was required. As a result, the definition of Low-Income households ranged from 153% of LICO to 225% of LICO, depending on household size. This percentage is shown in Table 3.1 under the LICO factor column. Household income data came from the Transportation Tomorrow Survey discussed later in the chapter. Large gaps between household sizes exist due to the aggregate nature of the survey data compared to the census; for example, there exists no income bin between 59,999 and 99,999 in the survey. In an ideal project, income data would be more disaggregated to reduce the large increases in the upper bound as household size increases.

The definition of equity seeking groups was not discrete, and individuals can belong to multiple equity seeking groups. As an example, a Black transit rider would also automatically be considered a Racialized rider, and a Carless Household can also be a Recent Immigrant household.

3.4 Determining Trip Patterns of Equity Seeking Groups

Many of the measures proposed require OD trip data that shows the number of transit trips from one traffic analysis zone (TAZ) to another. The 2016 Transportation Tomorrow Survey (TTS) fulfills some of these requirements.

The TTS is a travel survey conducted in the Golden Horseshoe region surrounding Toronto [28]. Every five years, approximately 5% of households were asked to describe all trips, on all modes, made by all members in that household in a travel diary [70]. The most recent version of the TTS was conducted in 2016, with results released in 2018. The TTS had the predicted number of OD trips made by the General Population among all TAZs in the Golden Horseshoe, including Toronto's 625 traffic analysis zones. While having fields relating to vehicle ownership status and income, the TTS lacks other demographic data such as immigration status and race.

To estimate the number of OD trips made by other equity seeking groups in Toronto, additional data from the 2016 Canadian Census was used to supplement the TTS data. Publicly available Census Dissemination Area (CDA) profiles were used [75], since disaggregate census data was not available for the study. The census profiles include data on the number of each gender that responded to each census question; as an example, the number of male and female Recent Immigrants living in a CDA can be identified from the census profiles. Since the TTS had information on the gender and household location of the transit commuter, census data can be matched to the TTS to provide an estimate of the number of people making a trip from an origin to a destination belonging to the following equity seeking groups:

- Recent Immigrants
- LEP
- Racialized
- Black Canadians

This approach takes into consideration that residents who reside at a specific zone may not always make trips starting in their home zone to another zone, or vice versa. For example, a Low-Income rider who lives in Scarborough may make regular trips from the CBD to Etobicoke.

CDA boundaries do not perfectly match TAZ boundaries; most CDAs in Toronto had populations ranging from 0 to 2,000 [75], while population in TAZs can range from 5,000 to 15,000 [27]. In cases where CDAs belong to two different TAZs, a constant population density was assumed, and the CDA data was split proportionally based on the area belonging to each TAZ. Results for a TAZ that contained multiple whole CDAs and split were created by summing the population for all CDAs and split CDAs.

This approach to synthesizing the number of trips made by Black, Recent Immigrants, LEP, and Racialized riders does not match the level of accuracy of collecting demographic information (such as income, or vehicle ownership) during the survey.

Origin Zone	**Destination Zone**	**Trips Made by All Transit Riders**	**Household Location**	**Period**	**Trips Made by Racialized Riders**
1	2	10	5	PM	5.0
1	2	30	1	PM	7.5
1	2	6	5	EM	3
1	2	3	1	EM	1.5

Table 3.2: Given parameters for simple demographic estimation example.

The estimation was subject to potential errors that may have occured during the collection of the TTS data. It also assumes that the proportion of each equity group living at a specific zone and making trips will not change by time of day. Future work could build on the methodology by introducing a more robust estimation process for the number of trips made by specific demographic groups, or by using a travel demand dataset that includes demographic information to avoid the need for estimation.

Table 3.2 presents data for a simplified and hypothetical example of how trips were estimated. For this example, 25% of residents living in zone 1 were Racialized, whereas 50% of residents living in zone 5 were Racialized.

In this example, 40 trips were made by transit riders in the afternoon rush hour between zone 1 and 2, while 9 trips were made in the early morning. For Racialized riders, 12.5 riders travelling between zone 1 and 2 in the afternoon rush hour was estimated, and 4.5 riders in the early morning

Figure 3.4 shows the origin and destinations of trips made by each equity seeking group in the AM period, while Figure 3.5 and Figure 3.6 shows the estimated number of transit riders belonging to each equity seeking group by period. The bivariate map in Figure 3.4 easily shows the origin destination trip flows, and the concentration of origins and destinations. An alternative simplified version of Figure 3.4 is presented in Figures 3.7, and 3.8. Figure 3.7 shows trip flows aggregated to each Toronto community district, with the CBD separated out from Inner City Toronto, while Figure 3.8 aggregates data for Scarborough, Etobicoke, and North York into the single "Suburb" category. For all periods, equity seeking groups had a lower proportion of trips starting or ending in the CBD than the General Population and were more likely to make suburb to suburb trips, or trips that stay inside a district. Low-Income and Carless Household trips were more likely to make trips during the midday and evenings, and most equity seeking groups were much more likely to make trips in the early morning than the General Population. Beyond these differences, the rest of the equity seeking groups had trip patterns very similar to the General Population, which can be explained by the fact that the majority of the General Population belongs to any one of the six equity seeking groups used for this project. However, due to the caveats mentioned above, the estimation should be verified and expanded on in future work.

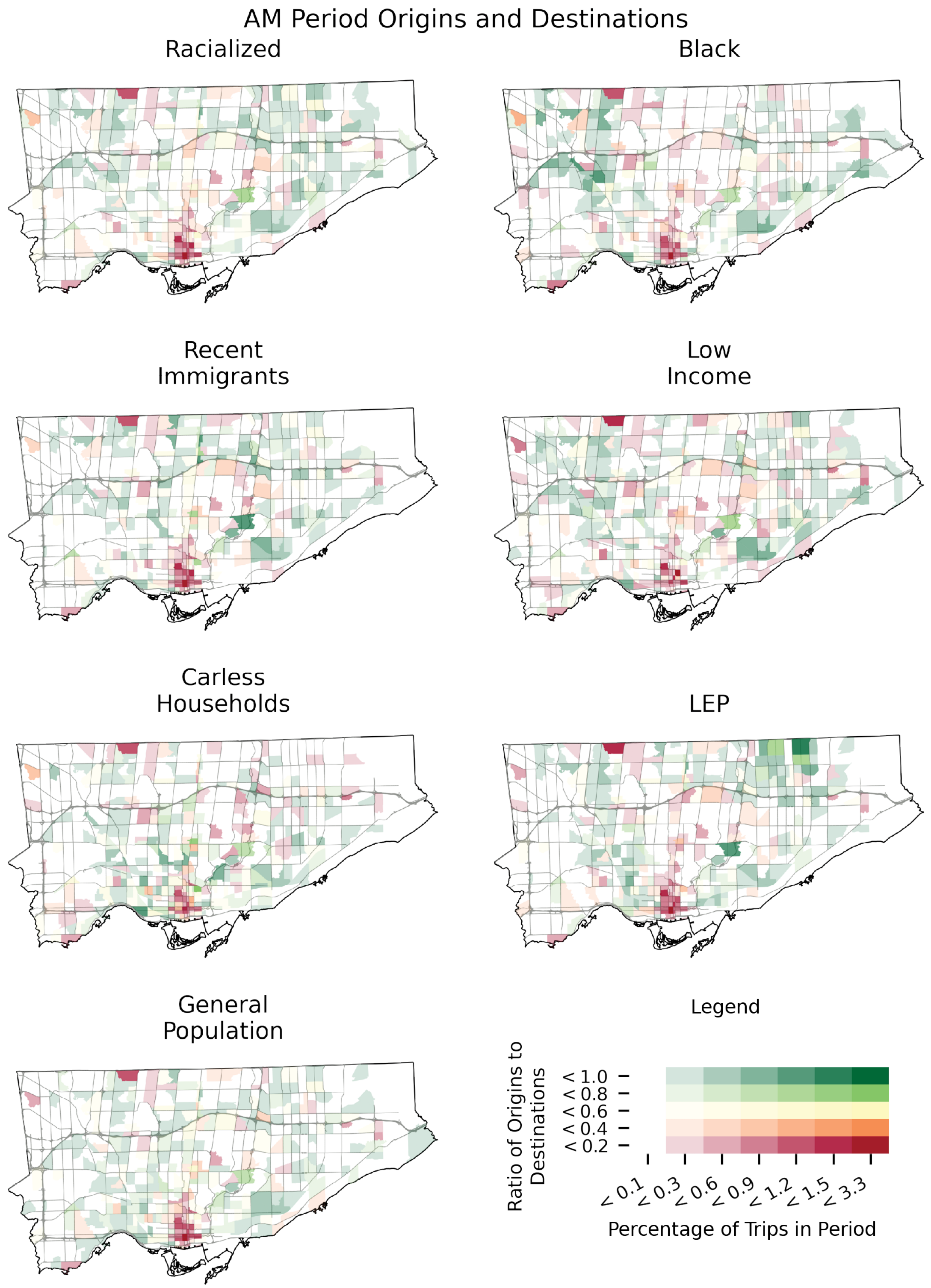

Figure 3.4: Bivariate OD Trip Patterns map for all equity seeking groups in the AM period. Maps representing other periods can be found in Appendix A. Darker shades of red and green indicate a greater proportion of trips starting or ending in a zone. Green indicates the TAZ contained more origins, while red indicates more destinations, and yellow indicates a roughly equal number of origins and destinations.

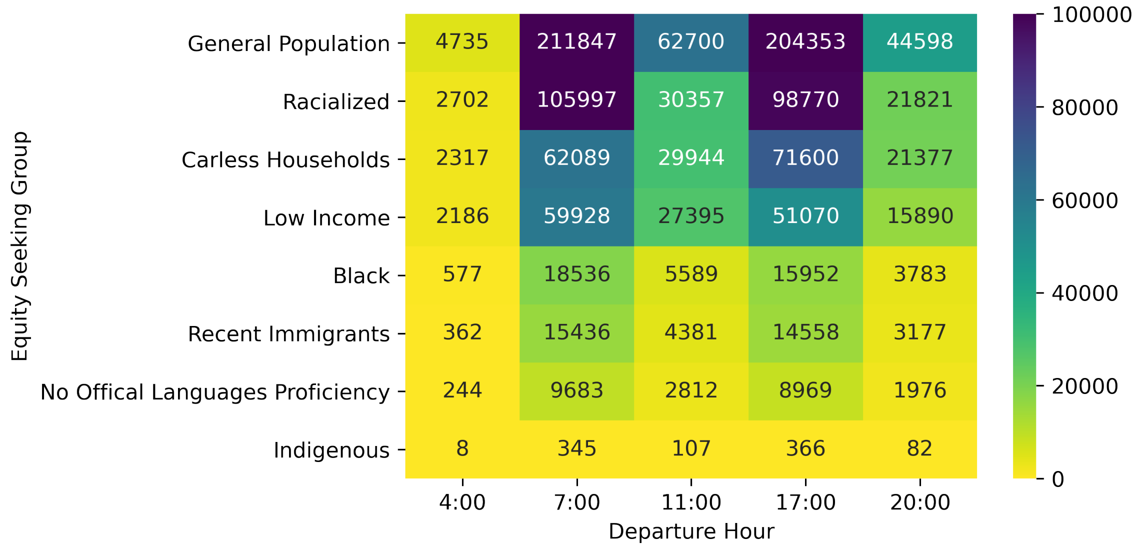

Figure 3.5: Number of individuals from each equity seeking group making a transit trip for each period.

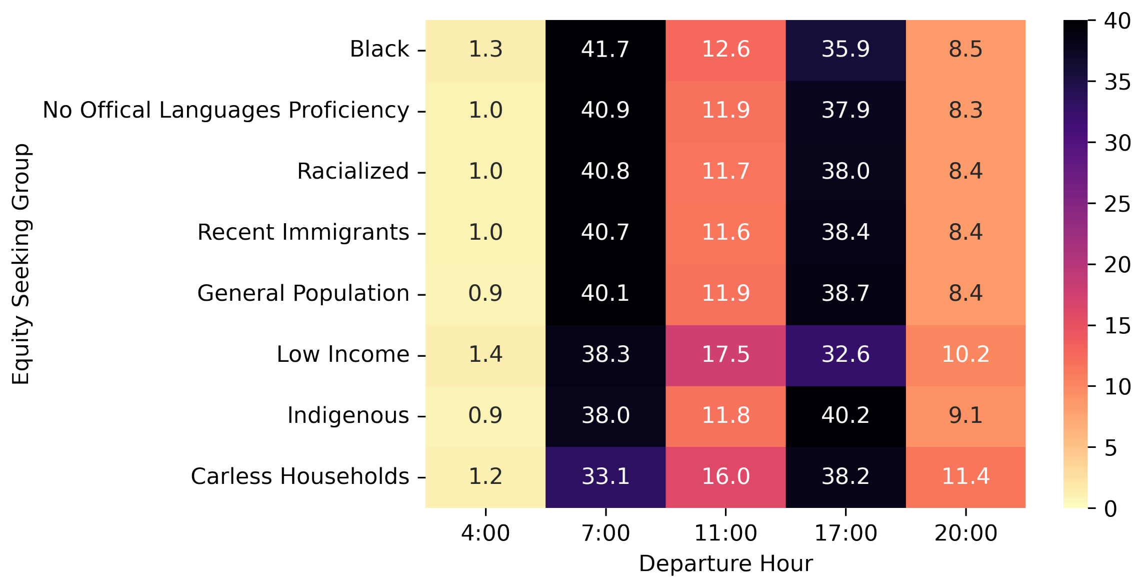

Figure 3.6: Proportion of trips from each equity seeking group making trips during each period.

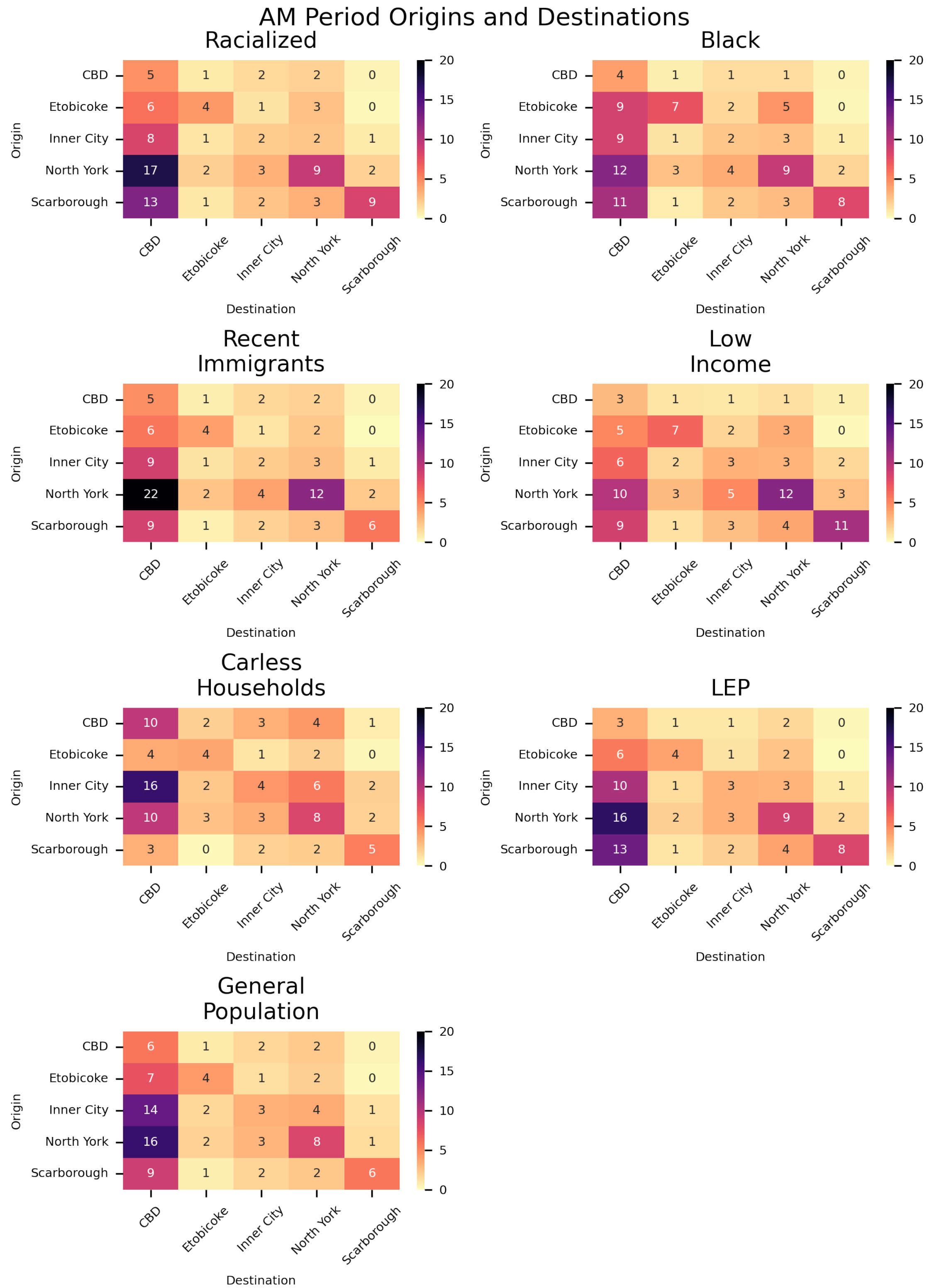

Figure 3.7: Percentage trip flows for the AM period, aggregated to the district level. Charts representing other periods can be found in Appendix A.

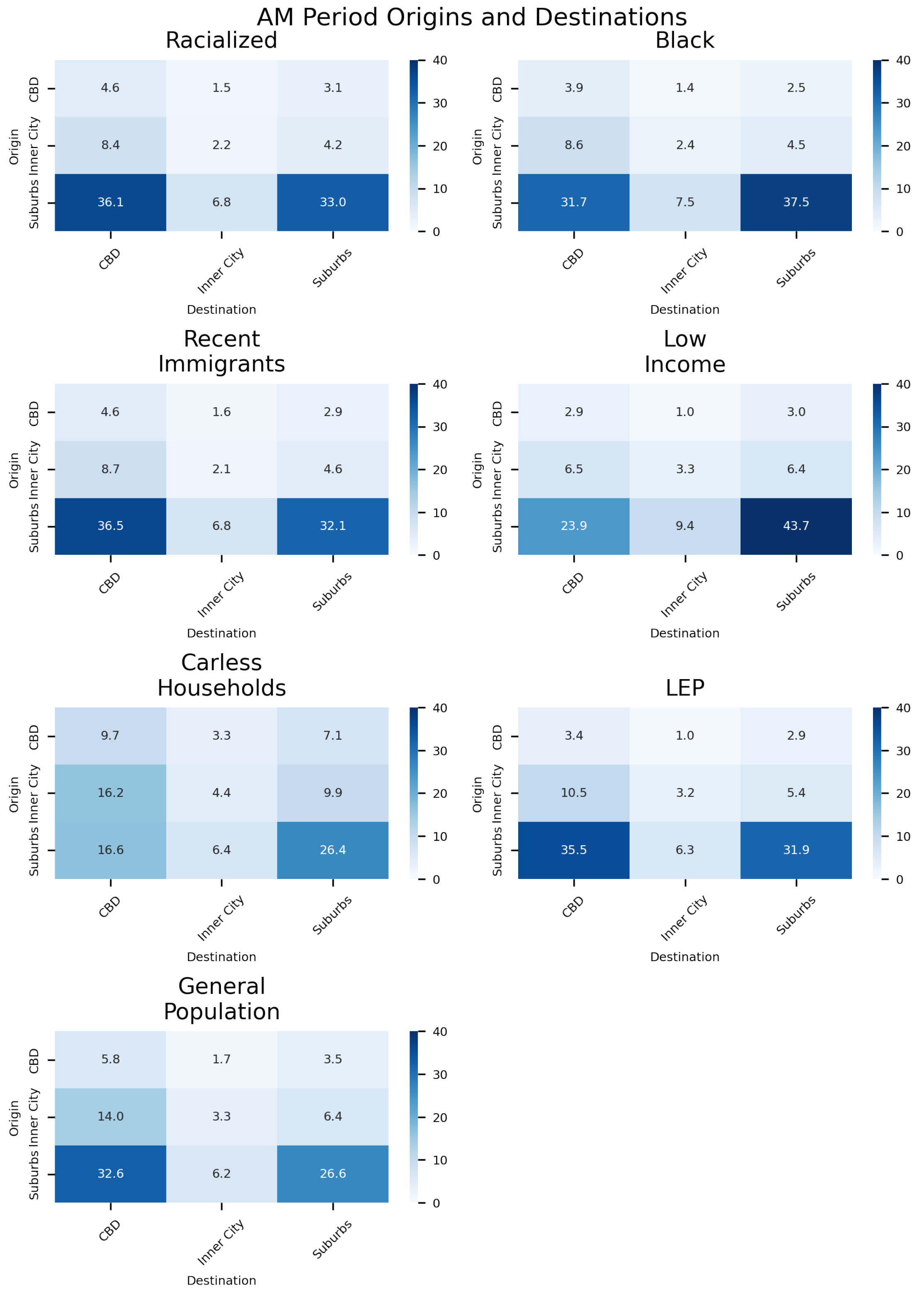

Figure 3.8: Percentage trip flows for the AM period, aggregated into the CBD, Inner City, and Suburb categories. Charts representing other periods can be found in Appendix A.

3.5 Route Map Graph Construction

For the route map graph or L-Space graph, the October 2016 version of the TTC's General Transit Feed Specification (GTFS) was used to provide the schedule of bus, streetcar, and subway service [83]. October was chosen to align the graph with the TTS collection period.

Instead of representing each bus/streetcar stop and subway platform as a discrete node, intersections and subway stations were represented as nodes. Directed edges would then be drawn between each intersection/station instead of between each stop. Only major and minor intersections, defined by the City of Toronto centreline dataset [19], were represented; this was done to reduce the number of nodes needed to represent the network. While not an issue for the route-map graph, the time-expanded graph uses the same set of nodes as the route-map graph, and additional nodes would cause performance issues. This decision will be discussed in the next chapter.

Stops located near an intersection were assigned to the nearest intersection. The criteria for which intersection a stop was assigned varied depending on the classification of the intersection. Intersections between two major arterial roads would have a larger catchment radius than an intersection between a collector road and minor arterial road. Table 3.3 shows the general criteria for which stops were assigned to intersections. Like the intersection classifications, road classifications were taken from the City of Toronto centreline dataset [22].

First Road Classification	**Second Road Classification**	**Catchment radius (m)**
Subway Station		120
Major Arterial	Major Arterial	100
Major Arterial	Minor Arterial	90
Minor Arterial	Minor Arterial	75
Major Arterial	Collector	70
Remaining Minor Intersections		50

Table 3.3: Catchment radius for each type of intersection.

Figure 3.9 shows two examples of how bus and streetcar stops were assigned to intersections. In all intersections, the assignment was manually reviewed using QGIS, and some stops outside the catchment radius were manually assigned to the intersection. The example on the left in Figure 3.9 shows certain streetcar stops belonging to Bingham loop at Victoria Park Avenue and Kingston Road were not located inside the 90m catchment radius. Since those stops serve the intersection, the catchment polygon was altered manually to include those stops.

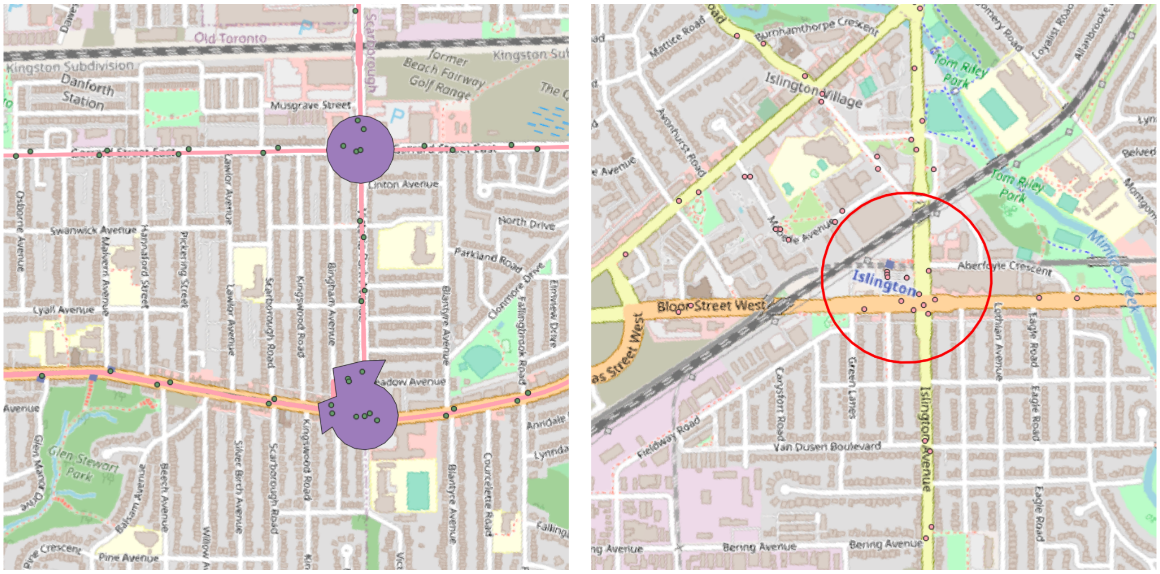

Figure 3.9: Examples of stop assignment to intersections. Left shows an example at Victoria Park Avenue, Kingston Road and Gerrard Street, while right shows an example at Islington Station.[64]

Because not every intersection was represented, and not every bus or streetcar stop was located at an intersection, 1364 out of the total 10,483 transit stops were not assigned to any intersection/station and were left out of the graph, with a large proportion of these stops being mid-block stops.

Using intersections/stations instead of stops as nodes simplified the graph by representing transfers without the need for excessive edges or nodes. For an intersection served by four stops, and two bus routes, only one node was needed to represent the intersection instead of four, and 12 edges representing walking connecting all four stops would not be drawn. This approach limits transfers to only occur at intersections or subway stations. It prevents an itinerary where a rider must walk 10 minutes between two intersections located further apart to transfer from one bus stop to another.

Edge costs, or travel time between two intersections, were determined by using the GTFS departure and arrival times. For each route, the GTFS stop time data were transformed into a feed that only included the departure and arrival times for each intersection/station that a route serves. For a route, if multiple bus stops serve an intersection, then the departure and arrival time for the stop with the closest distance to the intersection was used as the representative departure and arrival times for that intersection. This was often the case when there were both near-side and far-side bus stops at an intersection, or if there were stops on access roads leading to the subway station. The travel time on an edge was then taken as the difference between the arrival and departure times at the edge's downstream and upstream nodes, respectively. While the GTFS had departure and arrival times to the nearest second, the times used for the edge costs and the time-expanded graph were rounded to the nearest minute, making an edge cost of 0 minutes possible.

Netwokx [40] was chosen as the tool for the route-map graph analysis as it was based in python [87] and had many pre-existing algorithms. The connectivity measure required a multigraph, while the average neighbouring node degree measure was not compatible with multigraphs. Multigraphs are graphs where multiple edges can be drawn between the same pair of nodes. As a result, each individual trip segment between two intersections were added to the graph; therefore, the degree of each node represents the number of frequency/trips serving the node in one hour.

For the average neighbouring node degree, only one directed edge was drawn between two nodes. Each edge carries a weight representing a frequency factor, which was the sum of all frequencies using the road segment represented by the edge, normalized by the maximum frequency on any road segment in the entire network, similar to the approach used by Quintero-Cano [69].

3.6 Time-Expanded Graph Construction

The same process used to assign stops to intersections and determine the edge costs for the route-map graph was undertaken for the time-expanded graph. Additional performance and memory constraints for the time expanded graph meant that the number of nodes and edges must be optimized to prevent the graph from outputting an excessive amount of paths. The reason for the additional memory burden was the fact that a single node represents not only an intersection or station, but also an intersection or station at a particular state of time. For a time-expanded graph representing a one-hour period at a one-minute resolution, this meant an intersection/station was represented by 60 nodes, with each node corresponding to a different time in that hour. Edges were also time-dependent, since they would be drawn between two intersections whenever a transit trip exists to provide service between the two intersections.

Since the time-expanded graphs had a time dimension, they also had a length of time that defined the period of time the graph represents, as indicated in Figure 3.10, and a departure hour which represented the period trips must depart. For example, for a graph that represents the AM period, all paths had a departure time between 7:00 AM and 8:00 AM. The OD pairs used for analysis in the AM period were all trips in the TTS that departed between 7:00 AM and 8:00 AM. All route-segments that arrived at an intersection between 7:00 AM and 12:00 PM would be represented on the graph. A five hour length was chosen to ensure that all OD pairs in the TTS for a specific period would return a travel time; while the number of transit trips longer than 150 minute was rare, it was still possible depending on the random departure time and period. The aim was to balance memory considerations with the need for all OD pairs in the TTS to return a path and travel time. For shortest path algorithms, this implies that the longest shortest path that can be calculated by the graph was between 4 to 5 hours.

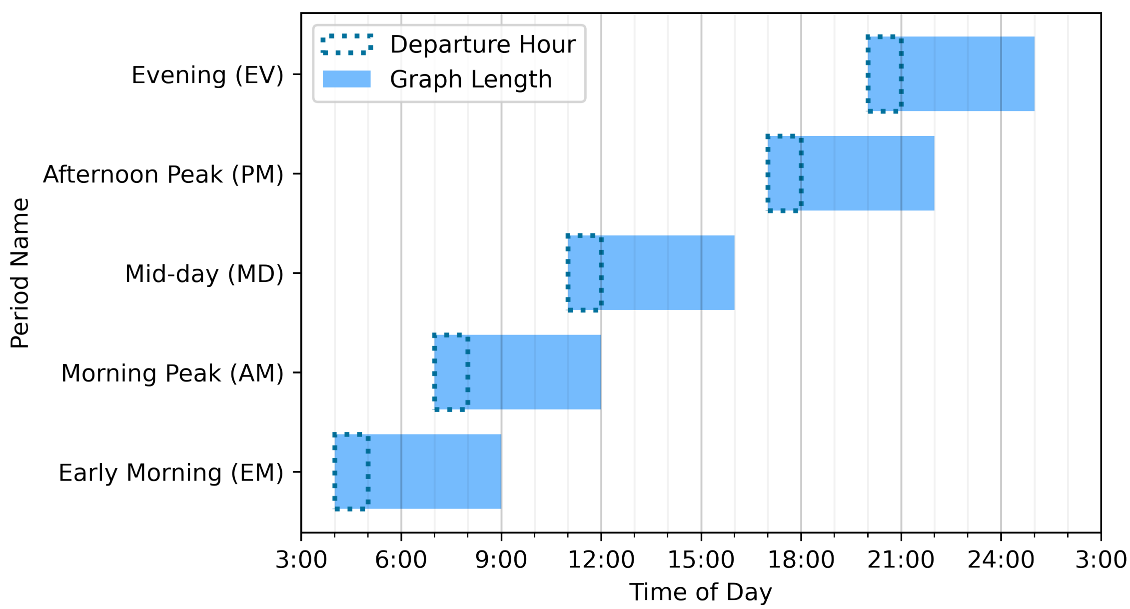

Figure 3.10: Time-expanded graph length for each period.

Unlike the route-map graph where a single node represents the intersection itself and transit stops at that intersection, two classes of nodes were needed to perform the same task.

The first class were index nodes, which represents the intersection itself at a particular state of time. Walking edges connecting it to TAZ centroids and boarding and alighting edges connecting it to route level nodes were drawn. Each index node had waiting edges that connected it to two other index nodes representing the same intersection: one representing the intersection one minute before and one representing the intersection one minute after. Times were rounded to the nearest minute instead of the nearest second, which reduced the number of nodes by a factor of 60 and reduced memory demand and computation time. The waiting edges and index nodes allowed a path to simulate the wait time at an intersection or subway station in between transit legs.

Route level nodes, which represent transit stops serving the intersection, were the second class. Each route level node represents service by one route in a single direction; most intersections were represented by at least two route level nodes. Besides being connected to index nodes with boarding/alighting edges, route edges representing the route serving the route level node, connected route level nodes to other route level nodes representing the intersection preceding and succeeding it along the route, at specific arrival and departure times based off the GTFS stop times data. Unlike index nodes, route level nodes only existed at times when a bus/subway/streetcar arrived at the station or stop. For example, for a route with a 10-minute headway, 12 route level nodes (6 per direction) were created to represent that route serving a specific intersection. Figure 3.11 shows the shortest path involving a transfer between two routes at an intersection, represented by both an L-space or route-map graph and a time-expanded graph.

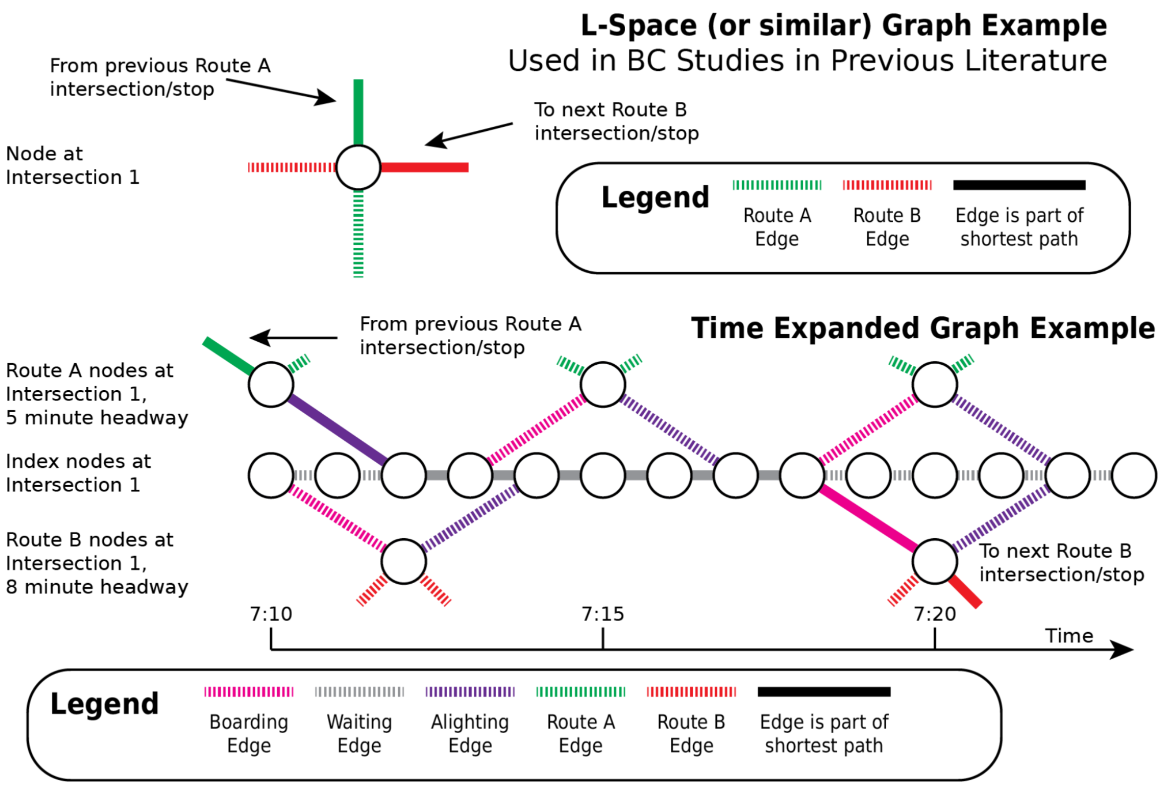

Figure 3.11: Representation of a path involving a transfer in an L-space graph and a time-expanded graph.

Boarding and alighting edges had an edge cost of two minutes. Together, both types create a minimum transfer time of four minutes. This simulated the time needed to walk between platforms at a subway station, or the time needed to cross a signalized intersection to transfer between two bus stops. These edges also reduced the potential for backtracking paths, especially in cases where long wait times at a station were involved. Eliminating backtracking improved the performance by reducing the number of extraneous paths, thus reducing memory demand of the graph.

Pathfinding graph measures, such as betweenness centrality, used TAZ centroids as the origin and destination. They were added to the graph as origin nodes and destination nodes. Like index nodes, origin nodes had a time associated with the node, and were represented at each minute of the graph. For example, for a 5-hour long graph, a single TAZ origin centroid was represented by 300 nodes, each representing a different time. For graph measures that calculated the shortest paths, destination nodes did not have a time associated with it, and each TAZ destination was only represented by one node. For graph measures that calculated paths with a set arrival time at the destination, such as the additional path factor measure which calculated the number of paths under a specific travel time threshold, TAZ destinations had a time associated with the node and 300 nodes was needed to represent a single TAZ destination centroid.

TAZs were connected by walking edges that represent the walk access time to and from the first or last transit stop. All stops located within a 15-minute walk of the TAZ centroid, or a 20-minute walk time for the EM period, were connected to the TAZ centroid, with some exceptions for more remote TAZs (such as Rouge Park, Toronto Island, and Cherry Beach). The edge costs for walking links represent the walk times connecting the TAZs and transit stops and were determined using OpenTripPlanner [65]. Since walking is not time dependent and does not have specific departure/arrival times, 300 walking edges were drawn, connecting a specific TAZ origin to a specific index node for a five-hour long graph, demonstrating that walking could occur at any point in the five hours a graph represents.

Some literature has shown that travel time can vary significantly based on the departure time [76, 35]. While not a huge concern for frequent subways or buses, the departure time can significantly change the travel time for paths that involve low frequency bus routes due to the transfer/waiting time required. Stepniak proposed having multiple departure times within several time bins, which was the approach used in this project. Graph measures that involved calculating shortest paths had six departure times for a specific period, with each departure time randomly found within a 10-minute bin. For example, for an analysis involving the AM period, departure times of 7:03, 7:10, 7:24, 7:38, 7:42, and 7:55 were randomly found, and the results for the six times were averaged.

Any measure involving a time-expanded graph required igraph [25] to compute the path itineraries and path lengths. Unlike networkx, igraph was implemented in C with a python API, and lacks the python memory overhead [68]. A 30x improvement in performance was reported by running algorithms in igraph as opposed to networkx, and graph analysis involving a single departure time would usually take 6 to 8 GB of memory in igraph while networkx would not load the time-expanded graph due to memory limitations. If a large number of paths needed to be calculated, then memory demand for all six departure times often rose above 200 GB, which required the use of virtual memory/swap from the solid-state drive. To facilitate multiprocessing and parallelization of tasks for the six departure times, a hexacore or octa-core processor was required for the project.

3.7 Real Time Network

Phase 3 of the project involved creating multiple time-expanded graphs, each representing a different period and day between November 28th, 2019, and January 3rd, 2020, excluding weekends and holidays. This period was chosen since the aim was to compare reliability and disruption for one scheduling period, or one GTFS feed. A winter scheduling period was sought since more weather-related disruptions occur during the winter. Real time data comes from two sources:

- Scraped subway arrival times from the TTC's Next Train Arrival (NTA) API [81]. The collection and preliminary cleaning of the data was conducted by Civic Tech Toronto [23], a volunteer-run organization dedicated to improving Toronto through technology.
- Scraped bus and streetcar arrival times from the TTC's NextBus API [82]. The collection and cleaning of the data was conducted by TransSee and Darwin O'Connor [85].

Neither the NTA nor the NextBus data included arrival times for Line 3 of the Toronto Subway. Static GTFS schedules were used for Line 3 [83], with certain trips removed based on whether a disruption or significant delay occurred. Wi-Fi data from BAI Communications [4] was used to determine which trips needed to be removed from the static GTFS feed. Most Toronto subway stations have platform level Wi-Fi access points, including 2 out of the 6 stations on Line 3. The Wi-Fi data can indicate when a train arrives at the station based on whether a significant number of devices dissociate from the access point. Line 3 maintains a headway ranging from 5 minutes to 6 minutes and 45 seconds depending on the period. If the gap between a train arrival and the previous train arrival was more than 7 minutes and 30 seconds, the corresponding trip was removed from the static GTFS. After removing trips, the arrival and departure times for Line 3 from the GTFS feed was added to the graph using the same process as the static time-expanded graph. Since the Wi-Fi data only included 2 out of the 6 stations on Line 3, the Wi-Fi data could not be used to determine arrival and departure times for the entire line.

The scraped NextBus API data was in a similar format as the GTFS feed and required almost no additional work to clean. Stop arrivals were already linked to the route, direction, and trip that encompassed that stop. The data was separated into files, with each of the 115 files representing one of 23 weekdays in the scheduling period, and one of 5 periods. Each individual file was then added to a graph using the same method as the static time-expanded graph.

On the other hand, the NTA data needed additional cleaning for further processing. Every minute, the next two to three train arrivals for each station and direction was predicted. Only the next train arrival in each direction was kept for the project. However, this still resulted in multiple arrival predictions for a train at any station since travel between successive stations often exceeds one minute, and headways were roughly two to five minutes depending on the line or period. To further eliminate multiple predictions, only the most recent prediction for each station-train combination was kept, as that prediction was most likely more accurate than a prediction made one minute before. Each arrival had data for the vehicle number and direction of travel; both pieces of data were used to create a journey for each train that describes the order of stations the train visited while in service. Trips were created by splitting each journey at the terminus and yard locations. Due to missing data, subway delays, and subway disruptions, large gaps in the data were observed. If there was a gap larger than 30 minutes, then the trip was split into two separate trips. At the termini, the next trip was determined to start at least 45 seconds after the previous trip ended. This was done to prevent instances where the next trip departed from a terminus before the previous trip for the same train arrived at the same terminus.

Once cleaning of the NTA data concluded, the NTA data was split into 115 files, each representing a different day-period combination, and then added to the graph using the same process as the static time-expanded graph.

3.8 Graph Measures for Importance and Criticality

Betweenness centrality and global efficiency are measures that measure criticality and were used as the basis for the graph analysis. While algorithms for both measures exist in igraph, the structure of the time expanded graph meant that BC and GE results by node would not be usable, since each individual node represents an intersection at one instance of time. In addition, both measures did not consider OD trip flows of the network. For example, two intersections may share the same BC value, but if one intersection was used by twice as many riders, then that intersection was more important to the network. Both measures were adapted for the time-expanded graph, and to consider the impact of passenger flows.

3.8.1 Population Weighted Betweenness Centrality

Betweenness centrality of a node measures the ratio of shortest paths passing through a that node, summed over all origin-destination pairs. This measures how important a node was to all trips made in the network [30]. A high BC for a station or intersection would indicate that if that station or intersection was removed or taken out of service, it would be very disruptive to the entire network, since a large number of paths rely on that node. To apply the measure to a time-expanded network, for each period and departure time, igraph outputted all shortest paths for all OD pairs traveled in a period. The shortest path output was a list of nodes visited. The list of nodes in each shortest path was converted to a list of intersections or subway stations visited, removing the time dimension. Some shortest paths included the same set and order of intersections as other paths but had slightly different arrival times at some intersections. Removing the time dimensions allowed the identification of duplicate paths, and these duplicate paths were then removed. This process was then repeated for all departure times, and all periods. This was to ensure the measure counts the number of shortest paths that pass through an intersection. PWBC results were then averaged across all six departure times in a period.

A population weight was also added to the measure to create a population weighted betweenness centrality. The weight was the total number of commuters for a specific group travelling between an origin and destination over the total number of commuters for that specific population group and period, found in Chapter 3.3.

$$PWBC(i) = \frac{\Sigma_{od} x_{od} * \frac{n_{od}(i)}{n_{od}}}{x(N-2)(N-1)} \tag{3.1}$$

Where

- $PWBC(i)$ = population weighted betweenness centrality for node i, for a specific population group and period
- x = total number of transit trips in a specific period, by a specific group
- x_{od} = total number of transit trips in a specific period, by a specific group, travelling between a specific OD pair

- n_{od} = number of shortest paths between an OD pair
- $n_{od}(i)$ = number of shortest paths between an OD pair crossing intersection or station i
- N = number of intersections and stations in the graph
- od = origin-destination pair

3.8.2 Population Weighted Relative Global Efficiency

Similar steps were taken to modify the global efficiency measure to incorporate a population weight. As traditionally defined, global efficiency is the sum of the inverse travel times, summed for all OD pairs, and measures the total delay or disruption in a system [50]. Compared to summing travel times, global efficiency considers cases where the travel time cannot be calculated. Removing edges from the graph randomly can indicate how the intersections being removed were important to the network by measuring the GE after removal compared to a GE without any edge removals. It can also be used as a measure of redundancy by demonstrating if the network was still able to connect passengers in a reasonable amount of time if many nodes were removed. For the time-expanded graph, if a route segment was removed from the graph, all route edges representing that route segment (at different states of time) were removed.

The inverse travel time for an OD pair was multiplied by the ratio of trips travelling on that OD pair, summed over all OD pairs, to create a population weighted global efficiency. All population weighted GE results with intersection/station removals were compared to a population weighted GE result with no intersection/station removal, to create a population weighted relative global efficiency (PWRGE) measure, shown in Equations 3.2 and 3.3. While the original GE formulation requires normalizing the measure by the size of the graph, this was not needed since all results were being compared to the original state of the network, instead of being compared to other networks in other cities.

$$PWGE = \Sigma_{od} \frac{x_{od}}{x} * \frac{1}{c_{od}} \tag{3.2}$$

Where

- $PWGE$ = population weighted global efficiency for a specific population group and period
- c_{od} = shortest path travel time for a specific origin destination pair

- x = total number of transit trips in a specific period, by a specific group
- x_{od} = total number of transit trips in a specific period, by a specific group, travelling between a specific OD pair
- od = origin-destination pair

$$PWRGE = \frac{PWGE_i}{PWGE_{baseline}} * 100\% \tag{3.3}$$

Where

- $PWGE$ = population weighted relative global efficiency for a specific population group and period
- $PWGE_i$ = the PWGE for iteration i, with intersections removed for a specific population group and period
- $PWGE_{baseline}$ = population weighted global efficiency for the unaltered baseline network, for a specific population group and period

3.9 Graph Measures for Redundancy

Connectivity and average neighbouring node degree are indicators that use the route-map graph. To calculate the measure, the graph was divided into 25 subgraphs, each representing a ward. Wards generally have the same population as other wards, and ward boundaries are the same boundaries used for provincial and federal electoral districts. Connectivity and average neighbouring node degree were then calculated for each of the subgraphs. Based on the origin and destination ward of trips in each time period, weighted averages of the connectivity and average neighbouring node degree values were determined for each population group of interest.

The number of viable paths measure required the time-expanded graph. The original measure [47] did not consider the effect of ridership or trip flows, so a population weight was applied to the measure much like the weight applied to betweenness centrality and global efficiency. In addition, paths returned would need to be modified in order to create an intersection/station path instead of a node path.

3.9.1 Connectivity

Connectivity is an indicator that measures how freely an individual can move around a network [32]. It does so by summing the number of transfer opportunities, defined by nodes being served by more than one route. Frequency was accounted for by summing the degree of each node multiplied by a factor that indicates if the node was served by more than one route. To account for road segments that were served by overlapping transit routes, "multiple edges" were subtracted from the measure. Equation 3.4 and 3.5 shows the formulation for connectivity.

$$v_{tr} = \Sigma_i[(d_i - 2)\frac{d_i}{2}] \tag{3.4}$$

Where

- v_{tr} = transfer possibilities
- d_i = degree of node i

$$\rho = \frac{v_{tr} - e_m}{v_t} \tag{3.5}$$

Where

- ρ = connectivity
- v_{tr} = transfer possibilities
- e_m = number of multiple edges in the graph
- v_t = number of nodes served by more than one route (transfer nodes)

3.9.2 Average Neighbouring Node Degree

The average neighbouring node degree measures redundancy by taking the average degree of all nodes connected to the node of interest [5]; this measures the availability of service at neighbouring nodes. For a node with a high average neighbouring node degree, if a disruption occurred at that node, transit users could walk to nearby intersections or stations and use routes serving those nearby intersections/stations, which demonstrates high redundancy. Compared to the clustering coefficient, the average neighbouring node degree does not count the edges connecting neighbouring nodes with other neighbouring nodes, making it more appropriate for a bus network. Road and transit networks rarely connect neighbouring intersections with other neighbouring intersections, so a clustering coefficient would be inappropriate for this context. Equation 3.6 shows how the measure was calculated at the ward level.

$$\overline{NN} = \frac{\sum_i \overline{nnd_i}}{N} \tag{3.6}$$

Where

- $\overline{NN}$ = average neighbouring node degree score for the ward
- $\overline{nnd_i}$ = average degree of the nodes neighbouring node i, weighted by the frequency factor
- N = Number of nodes in the subgraph or ward

3.9.3 Number of Viable Paths

Number of viable paths is based on a measure created by Jing et al. [47] which measures the number of effective shortest paths. Since some paths involve segments that overlap with other paths, the paths are not fully distinct nor redundant. This measure computes an effective number of shortest paths between an origin and destination based on the number and length of overlapping segments among paths for a specific OD pair. The original formulation used the length of edge to determine the degree of overlap, while the implementation for this project used the travel time of the overlapping segment. Before computing the measure, paths composed of nodes were converted to paths composed of intersections/stations. Duplicate paths were then removed, much like the process used for PWBC and PWRGE.

The measure was calculated to find the effective number of shortest paths and the effective number of viable paths. Viable paths differ from shortest paths in that viable paths include cases where the travel time was slightly longer than the shortest path travel time. In situations where the primary path is unavailable to a transit user, alternative path choices become important and usable if they still allow a transit user to arrive at their destination with only a small delay. If the only alternative paths available cause a significant travel time delay, then no redundancy exists for the commuter. Equation 3.7 shows the process for determining what was considered a viable path travel time, given a shortest path travel time. This process was partially constrained by memory limitations; paths involving a significant delay to the shortest path travel time would cause millions of paths to be generated and the system to crash due to a lack of memory. Therefore, a balanced approach that weighed memory limitations with a desire to consider paths longer than the shortest path travel time was chosen.

$$VPTT = \begin{cases} SPTT < 30 & SPTT + 5 \\ 30 \leq SPTT < 120 & SPTT + 5 + [(SPTT - 30) * \frac{5}{90} SPTT] \\ SPTT > 120 & SPTT + 10 \end{cases} \tag{3.7}$$

Where

- $SPTT$ = shortest path travel time
- $VPTT$ = viable path travel time

For this measure, destination TAZ centroid nodes had a time associated with the node in order to compute the path, unlike the PWBC and PWRGE measures. The time was simply the arrival time of the path after adding the viable path travel time to the departure time.

The effective number of viable paths was then divided by the effective number of shortest paths to create an additional path factor (APF). Similar to previous measures, the APF was weighted by the number of commuters travelling between an origin and destination for each population group. Equation 3.8 shows the process to create the APF metric.

$$APF = \frac{\Sigma_{od} x_{od} * \frac{ep^{od}_{viable}}{ep^{od}_{shortest}}}{x} \tag{3.8}$$

Where

- APF = additional path factor for a specific population group and period
- ep^{od}_{viable} = effective number of viable paths between an origin zone o, and a destination zone, in a specific period d
- $ep^{od}_{shortest}$ = effective number of shortest paths between an origin zone o, and a destination zone, in a specific period d
- x_{od} = number of trips between an origin zone o to a destination zone d, for a specific population group
- x = number of trips for a specific population group and period

3.10 Conclusion

This chapter discussed three points: an overview of trip patterns on the TTC and relevant background information on Toronto, steps to produce the three types of graphs (route-map, static time-expanded, real-time time-expanded) required for all three phases of the project, and graph measures used in the three phases of the project. Chapters 4, 5, and 6 will apply the graphs and measures mentioned in the methodology and discuss results of the three phases of the analysis.

Chapter 4

Evaluation of Node and Edge Importance Across the TTC Network

4.1 Introduction

Two measures were used to evaluate the importance of the TTC network elements: population weighted betweenness centrality (PWBC) and population weighted relative global efficiency (PWRGE). The goal of this chapter was to see if different population groups were more concentrated on critical intersections, stations, or route segments, indicating the extent of their reliance on infrastructure/routes at those intersections or stations. Disaggregating the results by mode was also done to determine if different modes were more critical to different equity seeking groups compared to other equity seeking groups and the General Population. This insight can show if different groups were vulnerable to different types of disruption.

4.2 Population Weighted Betweenness Centrality Analysis

4.2.1 Evaluating the Use of Multiple Departure Times

As mentioned in the methodology, PWBC values for six randomly selected departure times within each period were computed. The results were then averaged to account for the impact of frequency and headway in the path itineraries. A brief analysis on the variation in the PWBC results for the six randomly selected departure times within each period was undertaken as a first step in the analysis.

The coefficient of variation (CV) among the six PWBC results for each node was calculated, where the coefficient is defined by Equation 4.1:

$$CV_i = \frac{\sigma}{\mu} * 100 \tag{4.1}$$

Where

- CV_i = coefficient of variation of the PWBC, for node i and a specific population group and period combination
- σ = standard deviation of the six PWBC values, for a specific population group and period
- μ = mean of the six PWBC values, for a specific population group and period

The variation was plotted in Figure 4.1. Results were disaggregated by period, and transit mode.

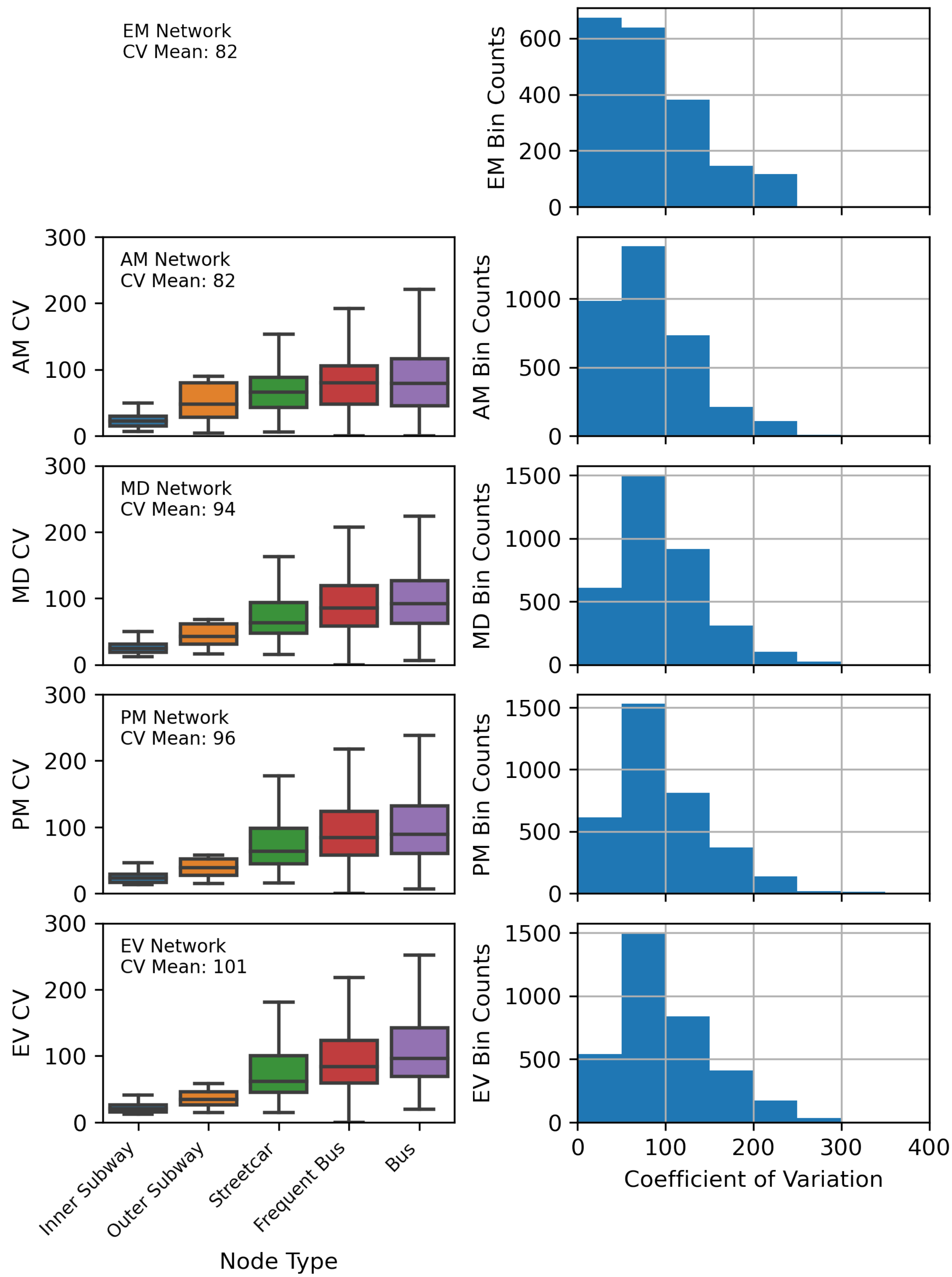

Figure 4.1: Box plot and histogram results of the PWBC coefficient of variation. Disaggregation by mode was not done for the EM period, since the subway was not operational during that period.

Very high CV was observed for all periods. While the mean was closer to 100%, which was already a high CV value, in some nodes the CV approached 400%.

Disaggregated by mode, the CV was higher for surface transit routes, such as streetcar or bus routes. Outer subway stations had a slightly higher CV, likely due to the longer headways of Line 3 and 4. Overall, this demonstrated the rationale for averaging results among six departure times, as the frequency of routes can influence the PWBC, and change the composition of the shortest path. This exercise also demonstrates the rationale for using a time-expanded graph compared to a route-map graph, especially for path analysis involving low frequency routes. Route-map graphs would not be able to consider the time dynamics associated with lower frequency surface transit routes and would cause greater uncertainty with the results since travel times and paths can vary.

4.2.2 Entropy and Top Ventile Mean of PWBC

The entropy of the PWBC was taken to evaluate the disparity of the PWBC among the nodes. A higher entropy would indicate that, for a specific population group, there would be a greater proportion of intersections and subway stations with very high and very low PWBC. The greater number of intersections and subway stations with high PWBC indicates that there was greater vulnerability for a significant transit disruption if those intersections or stations were taken out of service. The entropy was defined in Equation 4.2, with results shown in Figure 4.2:

$$E_{PWBC} = \Sigma_i \overline{PWBC_i} * \ln \overline{PWBC_i} \tag{4.2}$$

Where

- E_{PWBC} = Entropy of PWBC for a specific period and population group
- $\overline{PWBC_i}$ = PWBC for node i, averaged among the six departure times, for a specific population group and period

The PWBC mean was also calculated for nodes belonging to the top ventile. These nodes had the highest PWBC and would be the most critical to the entire network since more trips pass through these intersections and stations than lesser used nodes. This typically resulted in a set of 200-300 intersections, and the composition of intersections for the top ventile varied by population group and period. Except for the EM period, when the subway was not operational, all subway stations belong to the top ventile. The results disaggregated by period and population group are presented in Figure 4.3.

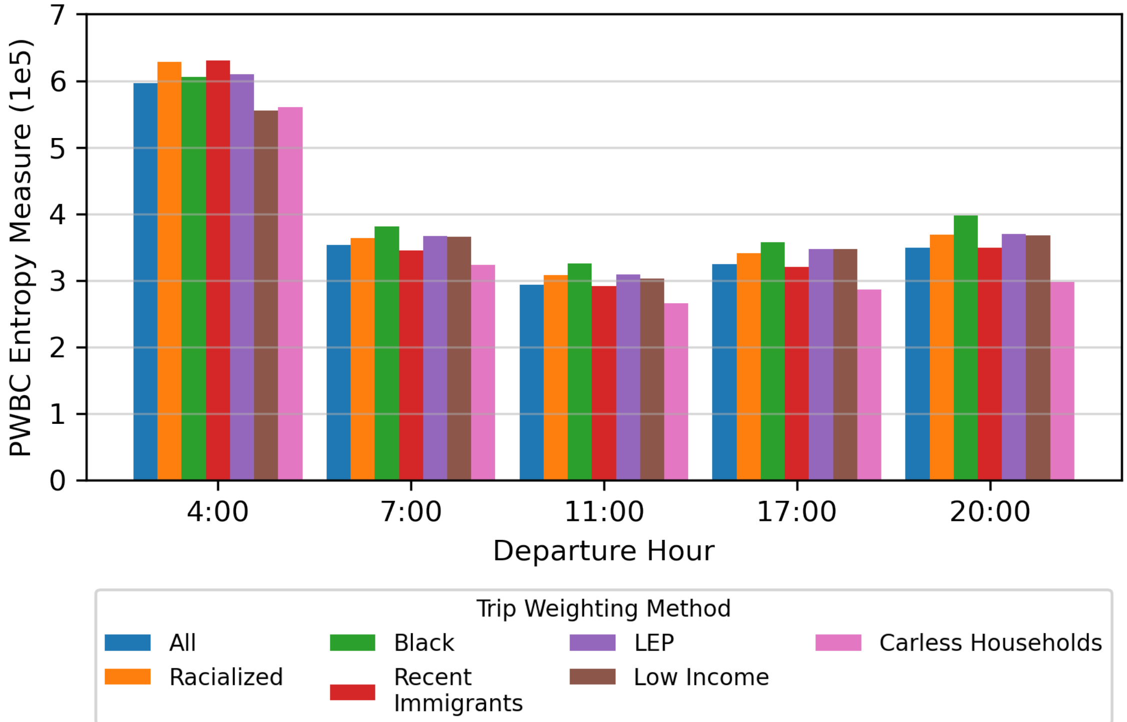

Figure 4.2: Entropy of PWBC disaggregated by period and population group.

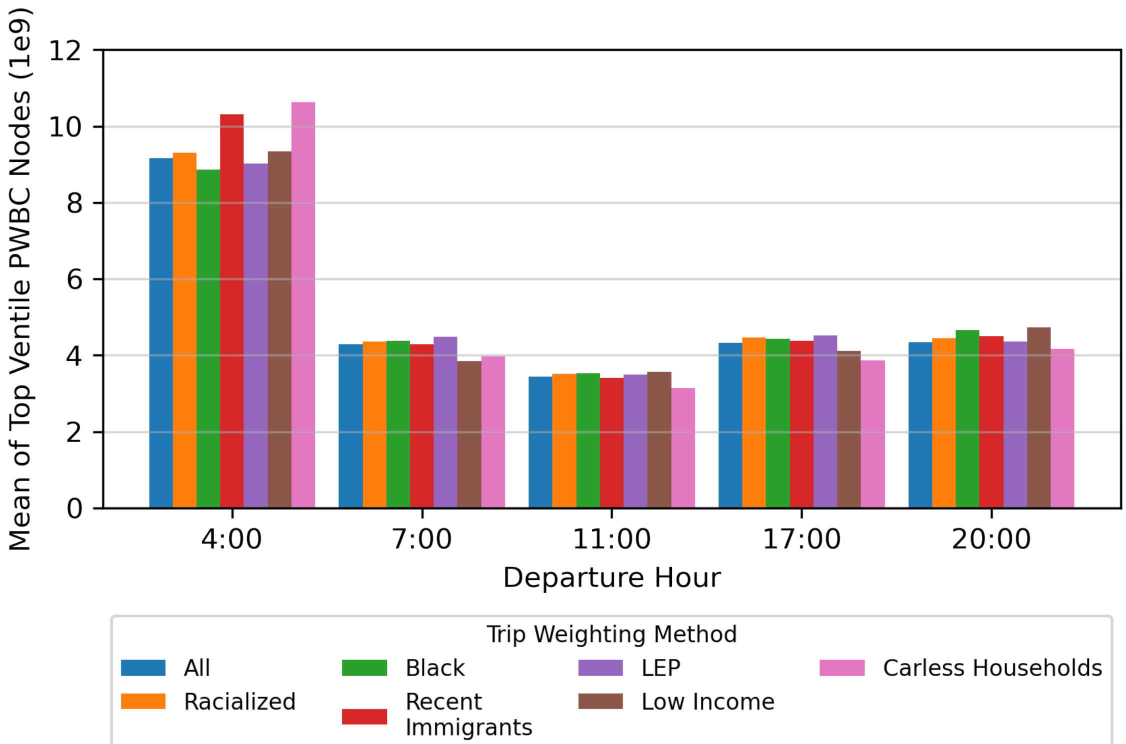

Figure 4.3: Top ventile mean of PWBC disaggregated by period and population group.

For nearly all equity seeking groups, the entropy of the PWBC was higher than the General Population, showing higher disparity among different intersections and subway stations. In addition, the top ventile mean shows that the most critical intersections for equity seeking groups tended to have a higher PWBC than the General Population. Both pieces of information indicated that trips made by equity seeking groups tended to be more clustered and concentrated around the most critical intersections and stations, while the General Population were slightly more distributed among different intersections and subway stations. Among the equity seeking groups, Black and LEP riders tended to be more concentrated, indicated by a higher entropy and slightly higher top ventile mean, while Carless Households tended to be the most distributed. If a disruption were to occur, by being less distributed, equity seeking groups would have a possibility of higher delay due to their slightly higher reliance on the most critical intersections and subway stations. In addition, due to their higher ridership numbers which can cause a higher possibility for security related incidents, the most critical intersections and stations would have a higher possibility of being taken out of service. However these differences were relatively small.

Additionally, the entropy of the PWBC and the mean of the top ventile PWBC was noticeably higher in the early morning period. Since Carless Households, Black, and Low-Income riders make 1.3 to 1.6 times more trips in the early morning period than the General Population, more of those riders were travelling in periods with higher vulnerability to disruptions.

4.3 Top Ventile PWBC Results Disaggregated By Mode

PWBC was also calculated for each edge, and the results were further disaggregated by transit mode. The proportion of edges in each mode that appear in each equity group's top ventile of PWBC scores appear in Figure 4.4.

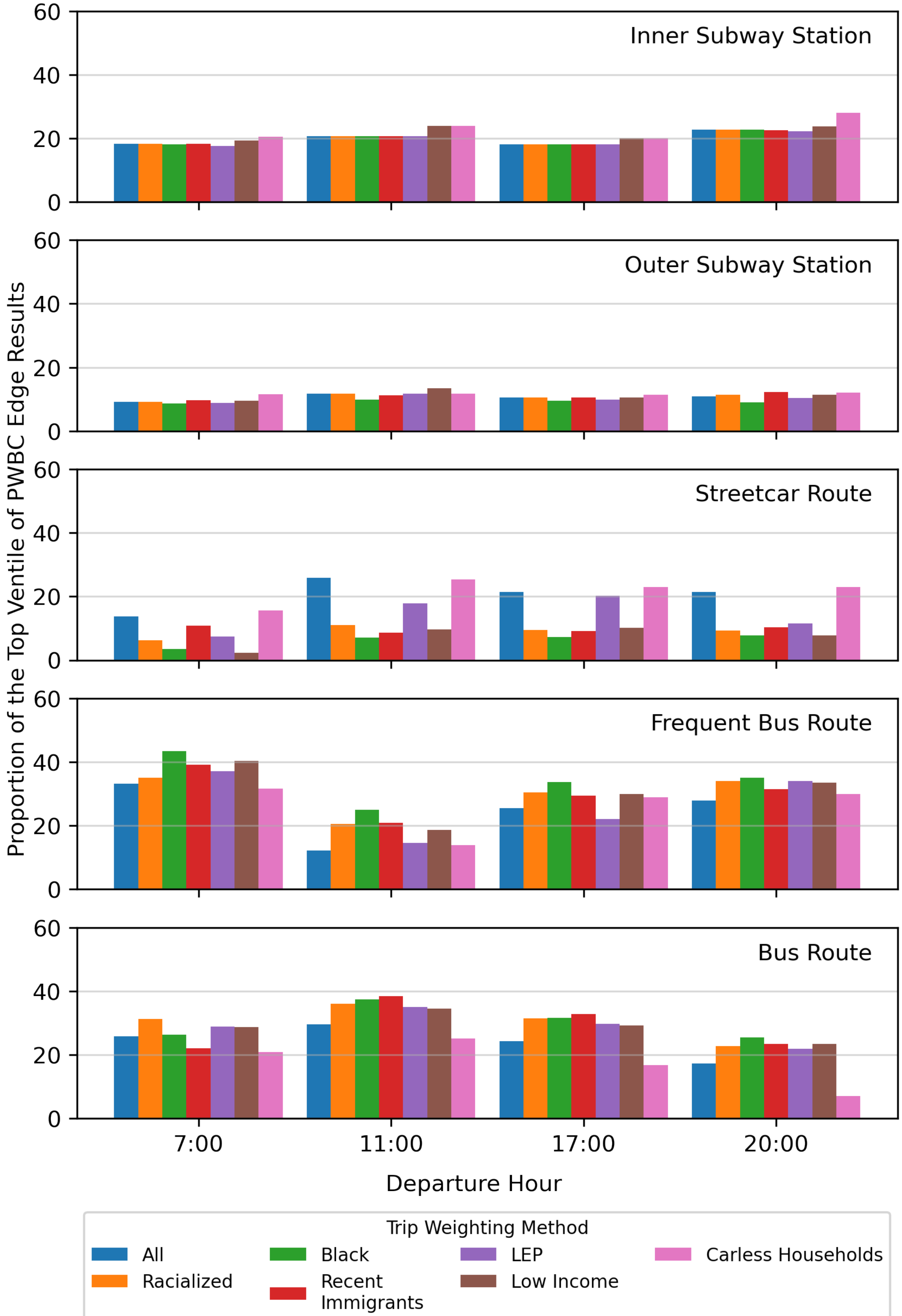

Figure 4.4: Top ventile PWBC mean by population group, period, and mode.

For equity seeking groups, a greater proportion of critical edges tended to be bus routes and frequent bus routes, showing the importance and criticality of those modes on the travel patterns of equity seeking groups. On the other hand, a greater proportion of critical edges for the General Population, Carless Households, and LEP riders were streetcar routes. These three groups were more likely to live or work downtown as the streetcar network serves downtown Toronto.

Carless Households and Low-Income households also had a greater proportion of inner subway stations in their top ventile of PWBC scores, compared to other equity seeking groups. Otherwise, the proportion of top ventile of PWBC results that were inner subway stations were similar for other groups.

Among equity-seeking groups, Black commuters had the greatest proportion of frequent bus routes, and a high proportion of bus routes, comprising its top ventile of PWBC, which shows the high importance and criticality of the bus network to Black riders.

Disaggregating by mode shows that different population groups had differing levels of importance and criticality for each transit mode, and disruptions would not have the same effect depending on mode. An equitable approach to disruption mitigation would be to minimize the probability of disruption on the bus network, as it would limit the vulnerability to disruptions and the negative impacts of disruptions for equity seeking groups compared to minimizing disruptions on either the streetcar or rail network.

4.4 Population Weighted Global Efficiency Analysis

4.4.1 Baseline Population Weighted Global Efficiency

The baseline population weighted global efficiency (PWGE) was calculated for all groups and periods. Ratios were calculated between the PWGE for each equity seeking group, and the PWGE for the General Population, shown in Figure 4.5. The aim was to show initial service and travel time discrepancies among equity seeking groups. Higher PWGE than the General Population indicates that the equity seeking group has a lower total travel time than the General Population.

The 2016 TTC network shows that Carless Households had a higher PWGE than the General Population, while Black, LEP, and Racialized riders had a lower PWGE than the General Population. Further chapters will explore the changes to the PWGE as a result of randomized edge removals.

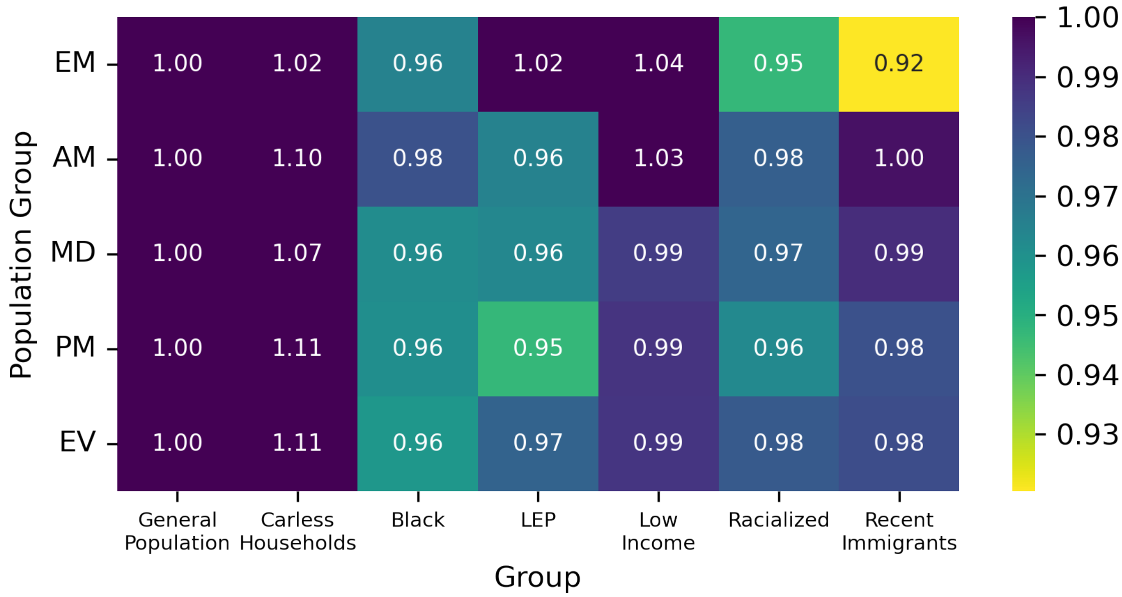

Figure 4.5: Ratio of PWGE between each equity seeking group, and the General Population, before any modifications were made to the 2016 TTC network.

4.4.2 PWRGE With Random Removals

To start the global efficiency analysis, route edges were randomly removed from the graph. After each removal, the population weighted GE was calculated for each of the six departure times in a period, averaged over all departure times in a period, and finally compared to the original GE with no removals to create the PWRGE. Edges were removed cumulatively; as edges were removed randomly during each iteration of removals, they were removed from all future iterations. This process was done for all periods, and all population groups. The results are shown in Figure 4.6, Figure 4.7, and Figure 4.8.

Groups with higher PWRGE would tend to be more distributed in their trip patterns, since a single route edge removal would not cause a detrimental increase in their travel times. This would indicate that individual route edges would be less critical and important. Besides indicating if edges were important, PWRGE also indicated which group would have the best redundancy and lowest delays in the event of a disruption; commuters belonging to groups with high PWRGE would be able to find alternative options.

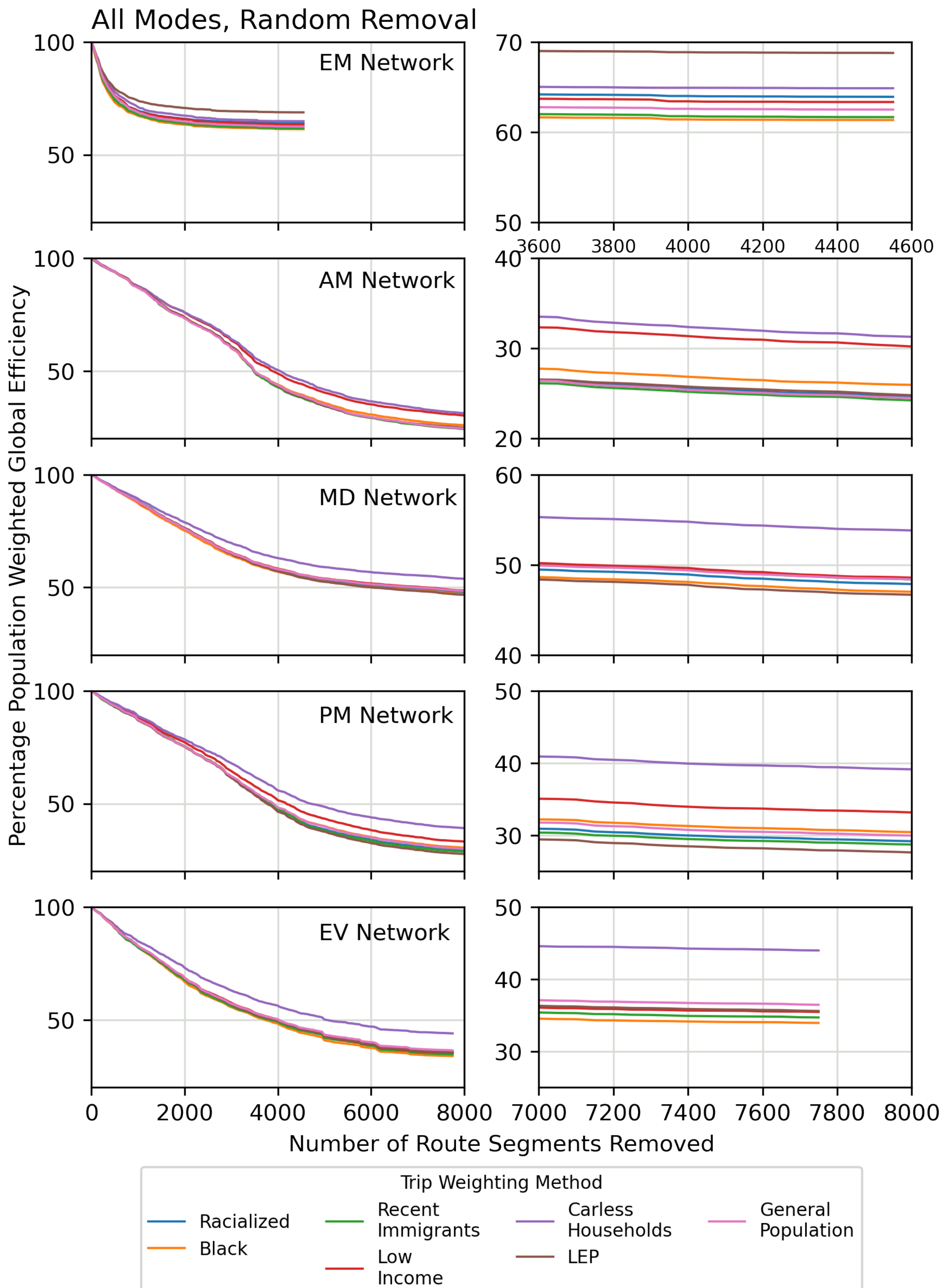

Figure 4.6: PWRGE for all periods and population groups in line graph form. The graphs on the left show results for all removals, while the graph on the right is a subset of the graphs on the left and show results between the 7000th removed edge and the 8000th removed edge, except for the EM period. As there are only 4600 edges in the EM period, the subset shows results between the 3600th removed edge and the 4600th removed edge.

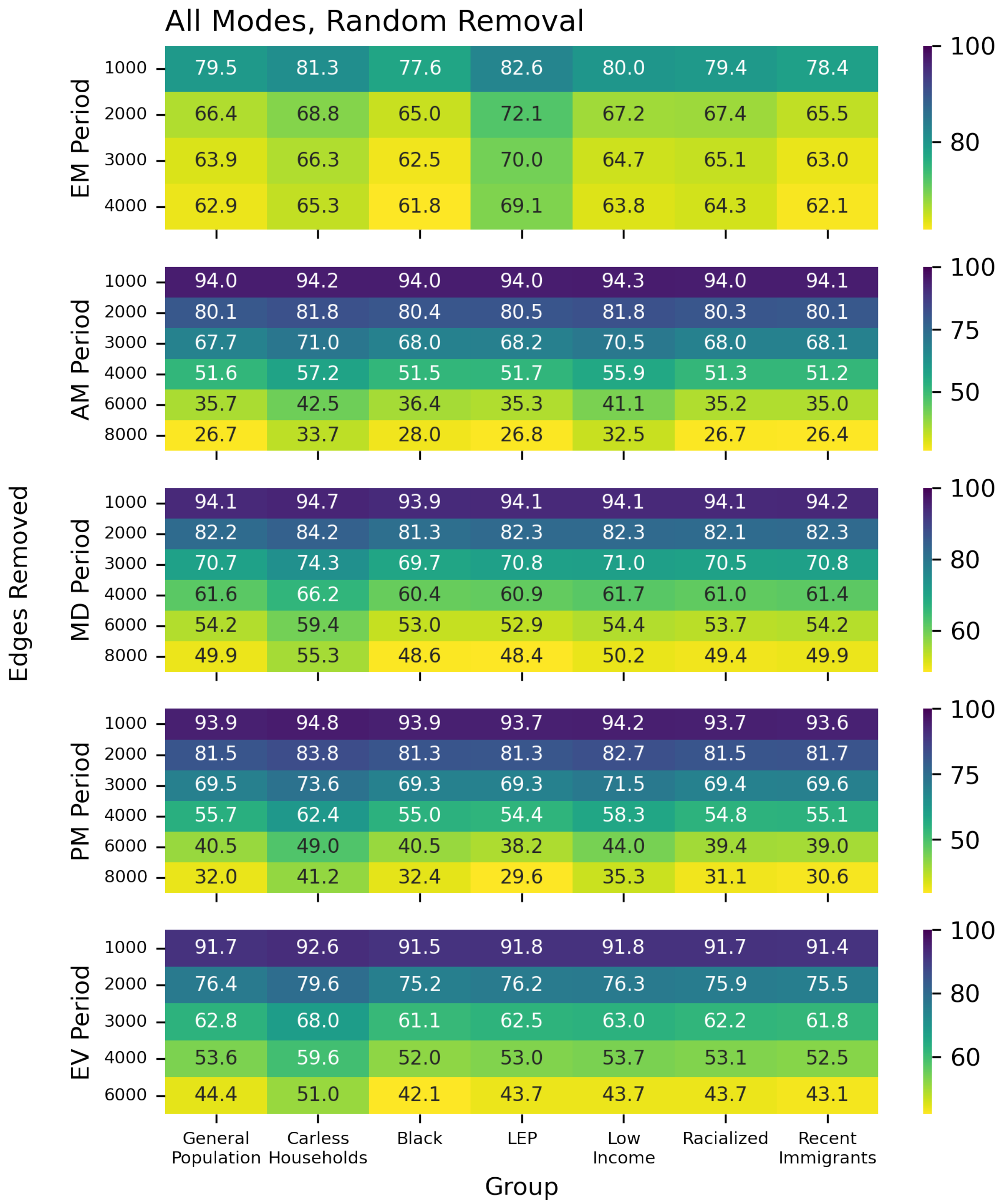

Figure 4.7: PWRGE for all periods and population groups in heatmap form. The results shown are the same as the results in Figure 4.6.

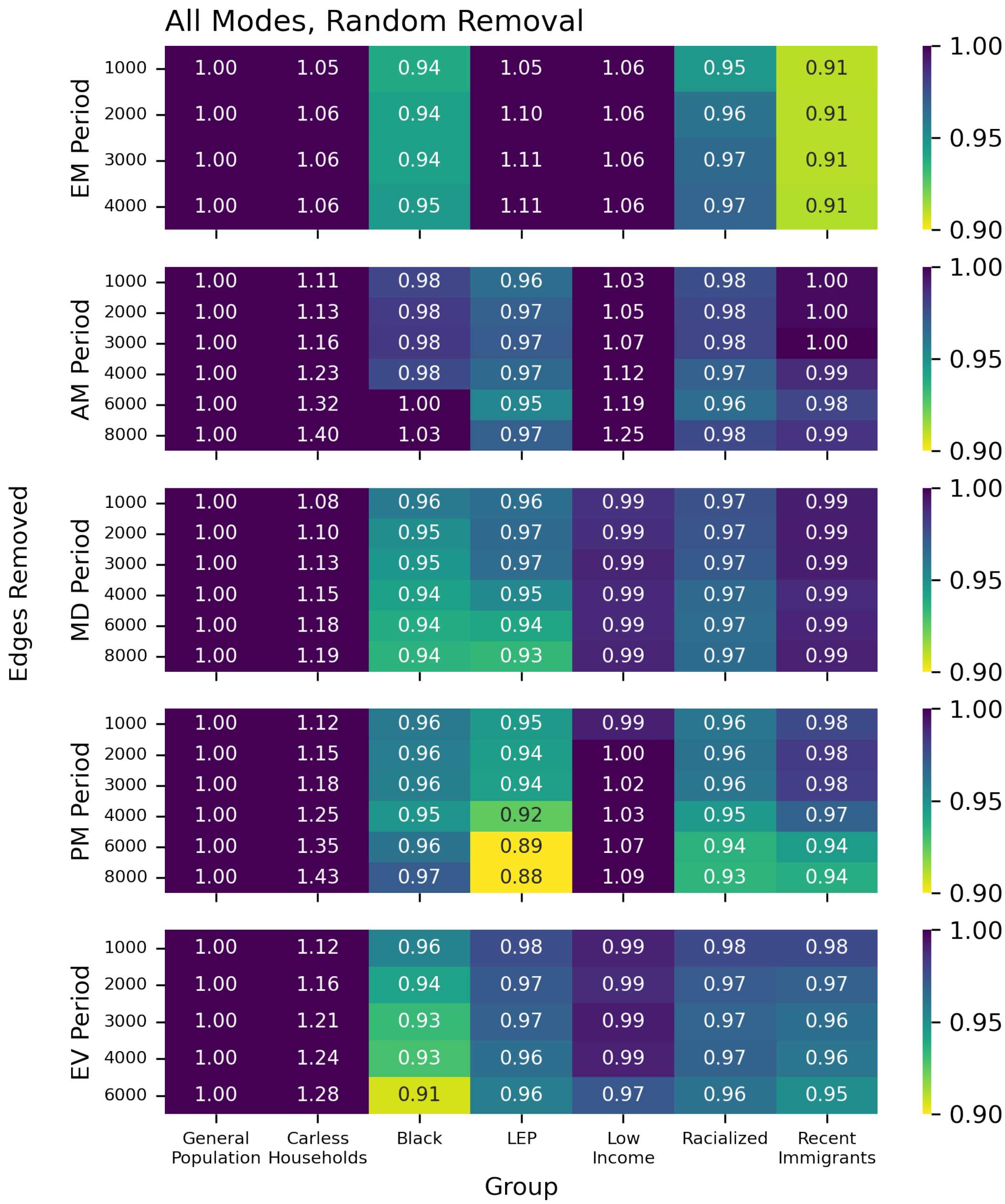

Figure 4.8: Ratio between the equity seeking group PWGE and the General Population PWGE for all periods and groups, as edges were removed.

While the differences between population groups were not large, they were consistent. The differences between population groups rarely shifted based on the period or the number of successive edges removed.

In general, Carless Households were less affected by random route edge removals, meaning that their travel was less affected by disruption, and transit was less critical to them. Many of their trips reported in the TTS were within walking distance, so the effects of the disruptions on Carless Households was mitigated since the time expanded graph was somewhat able to compute travel times for paths that involve only walking. This reinforces the observation that Carless Households were less exposed to disruption if individual intersections/route segments were disrupted.

Low-Income households had slightly better PWRGE than the General Population and other equity seeking groups, especially during peak periods. This may be because Low-Income riders during the peak periods, and Carless Households for all four daytime periods, had a lower proportion of CBD-based trips then other groups. Most CBD trips had alternative options that had much worse travel times, which may contribute to a lower PWRGE, while alternative paths for non-suburb trips may have similar travel times to the shortest path.

The early morning period was somewhat distinct from other periods. While Carless Households had a higher PWRGE than most groups, LEP has the best PWRGE among all population groups. Recent Immigrants and Black riders had the worst PWRGE during the early morning. The period also has the sharpest drop off in PWRGE in the first 1000 removed edges. This shows that groups that had a higher proportion of trips in the early morning period were slightly more exposed to periods with low redundancy and an increased potential for negative impacts from disruptions.

For the midday network, the PWRGE deterioration was lower than the other three daytime periods. Service drawdown in the weekday midday was not as severe compared to the evening, meaning that service headways were somewhat comparable to the morning and afternoon rush hours. Unlike the two peak periods, trip patterns in the midday were more distributed; Chapter 3 has shown the largest proportion of trips in the midday for all groups were suburb to suburb trips. This explains why edge removals may not lead to as much of a PWRGE drop compared to other periods.

Figure 4.8 shows that the differences in the PWGE between each equity seeking group and the General Population stay somewhat similar, with some exceptions. The gap between Carless Households and the General Population, Low-Income riders in the AM period, Black riders in the EV period, and LEP riders in the PM period widened, with Black and LEP riders having progressively lower PWGE than the General Population as the network was more disrupted. Low-Income riders continued to have a higher PWGE than the General Population in the AM period as a result of their suburban trip patterns. Given that Black, Racialized, LEP riders already start with a lower PWGE than the General Population, the ratios in Figure 4.8 show that those groups would continue to have a worse ability to travel around the network as the network becomes more disrupted.

As with the PWBC measure, differences among other population groups remain somewhat small; Black, Racialized commuters, Recent Immigrants, and LEP (in certain periods) still face slightly worse disruption impacts as a result of being more concentrated in their trip patterns and showing less redundancy.

4.4.3 PWRGE Disaggregated by Mode

As with the PWBC measure, PWRGE were disaggregated by mode. Route edges were not randomly removed, instead they were removed in order of highest PWBC for the General Population to lowest PWBC for the General Population. Heatmap versions of the results are shown in the main body, while line graph versions of the results are shown in Appendix B.

Figure 4.9 shows results for bus routes, while Figure 4.10 shows results for frequent bus routes. For the EM period, these two figures mirror Figures 4.6 and 4.7 since most routes in operation during those periods were bus routes.

For other periods, Black and Low-Income commuters consistently had the lowest PWRGE among all population groups, while Carless Households and the General Population had the highest. As with the results for random removals, Carless Households and the General Population had a higher PWRGE than other population groups, with the General Population performing on par with Carless Households for frequent bus routes. The high criticality of the bus network for Black and Low-Income riders may be caused by the higher proportion of suburb-to-suburb trips, and intra-suburb trips.

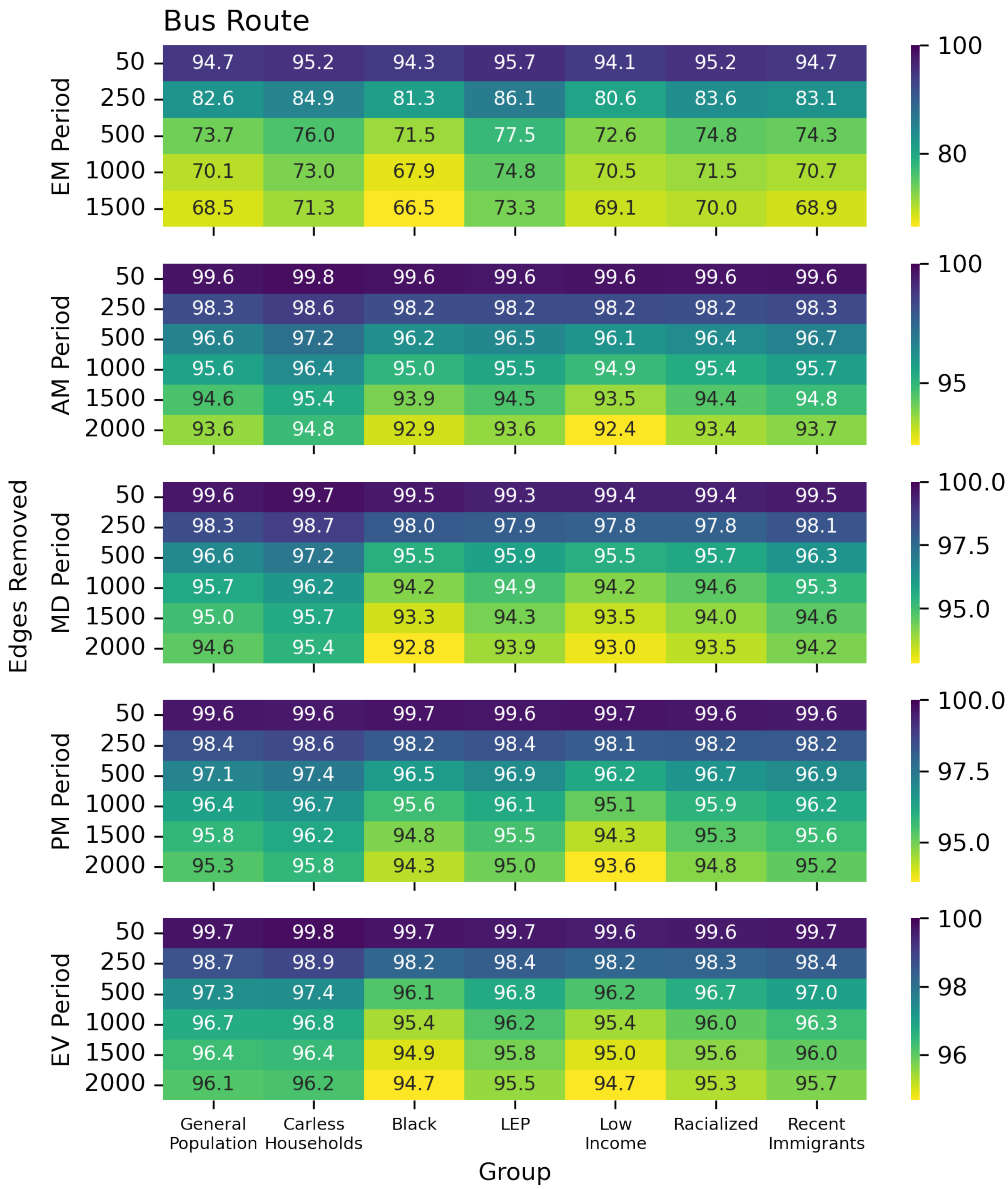

Figure 4.9: PWRGE results after removing edges representing bus routes.

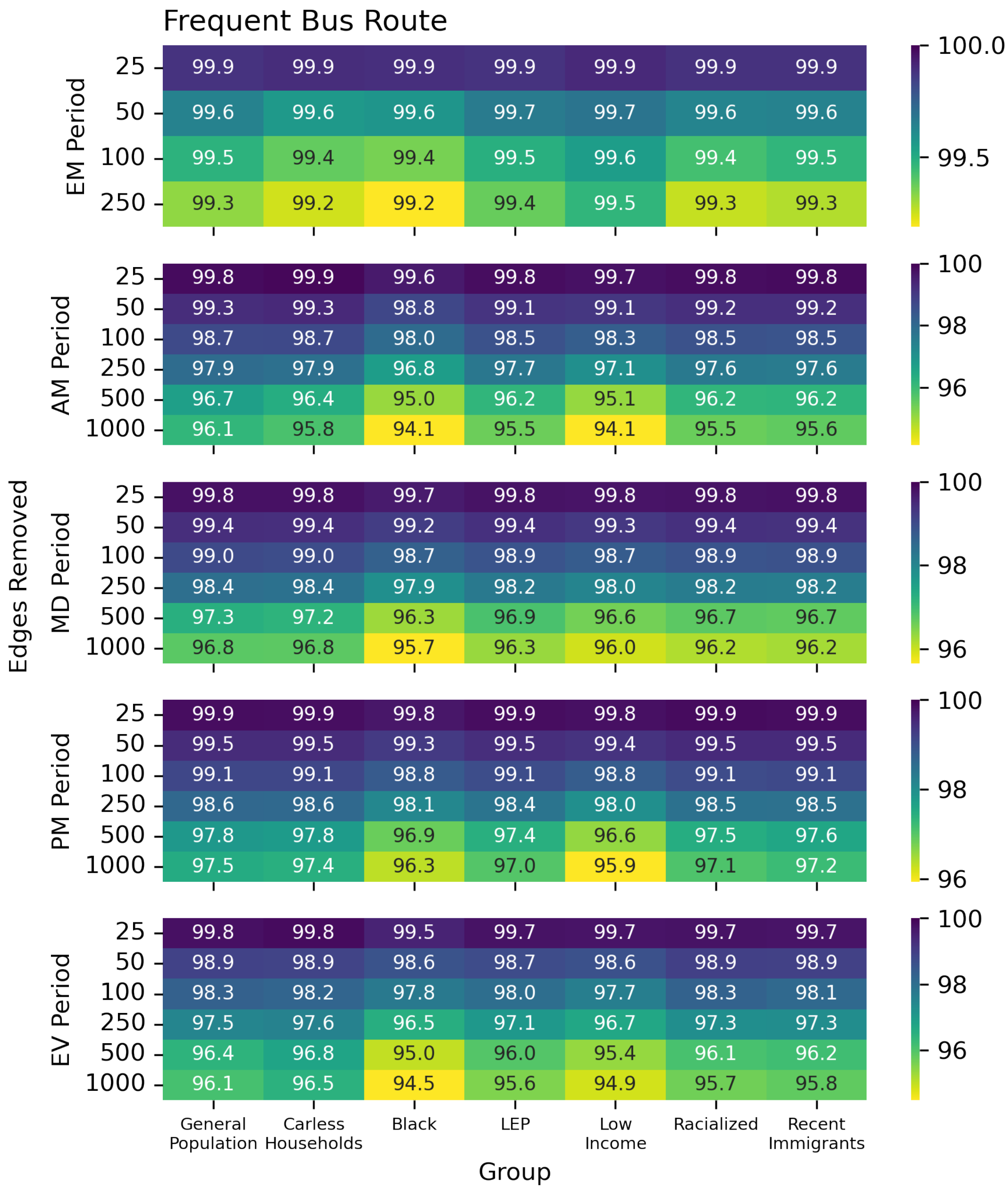

Figure 4.10: PWRGE results after removing edges representing frequent bus routes.

Figure 4.11 displays results after successively removing edges representing streetcar routes. In contrast to frequent and regular bus routes, Carless Households were the most affected by the removal of streetcar routes, followed by LEP. Both groups had trips concentrated in the CBD (and for Carless Households, Inner City Toronto), which would explain why the streetcar network has higher importance for those two groups, since the streetcar network mainly serves the CBD and the Inner City.

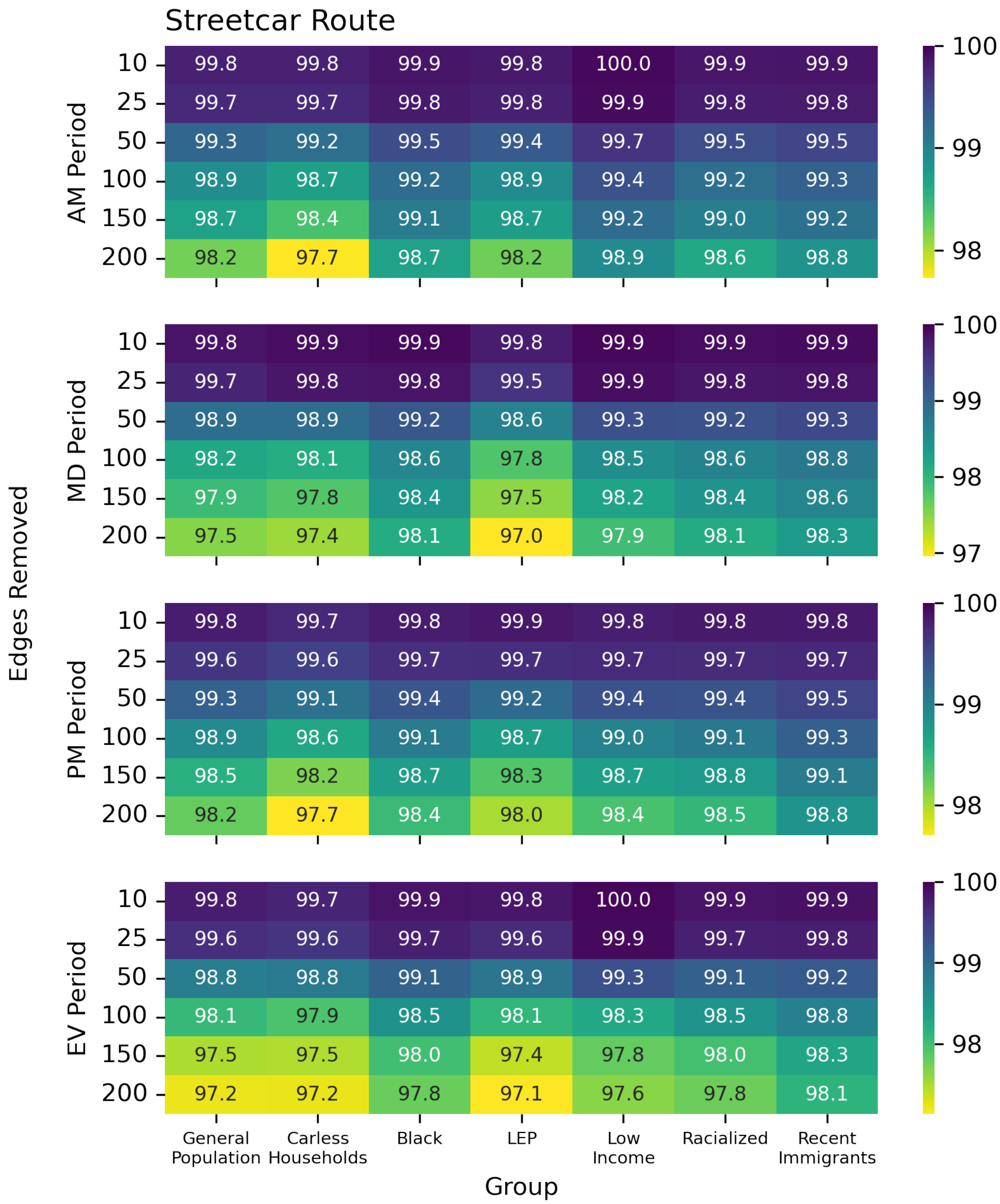

Figure 4.11: PWRGE results after removing edges representing streetcar routes.

Figures 4.12 and 4.13 displays PWRGE results after removal of edges connecting inner subway stations, and outer subway stations respectively. For inner subway stations, the General Population and Recent Immigrants were the most affected by the edge removals, with Low-Income commuters being the least affected. Other equity seeking groups had a slightly higher PWRGE than the General Population and Recent Immigrants. Low-Income riders have a higher proportion of suburban trips for all periods, therefore there would be less paths from Low-Income riders using the inner subway stations.

For outer subway stations, Recent Immigrants were still the most affected by the edge removals, while Carless Households were affected the least. This correlates with the higher proportion of North York-CBD trips this group has shown in their travel patterns.

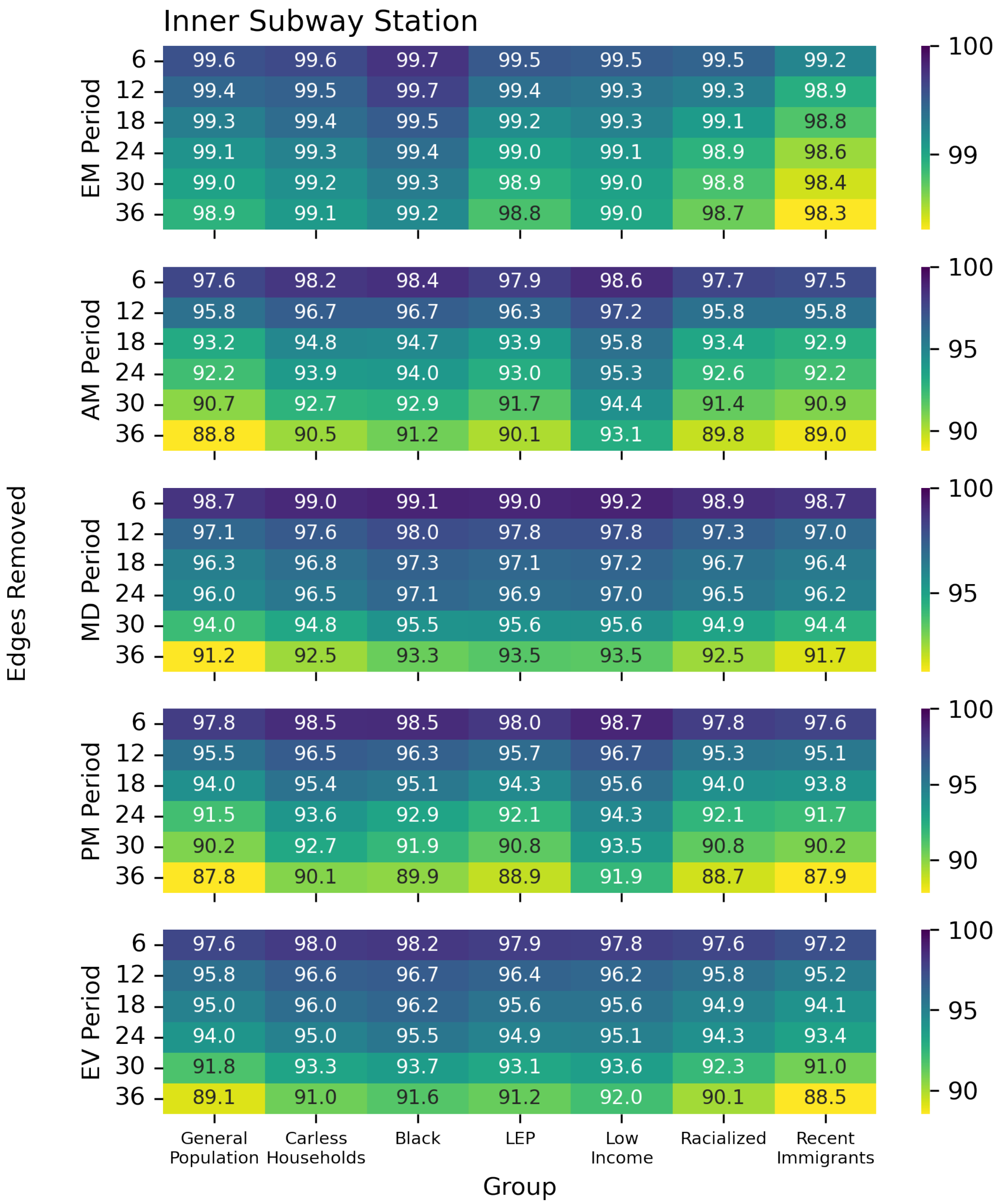

Figure 4.12: PWRGE results after removing edges representing subway segments connecting inner subway stations.

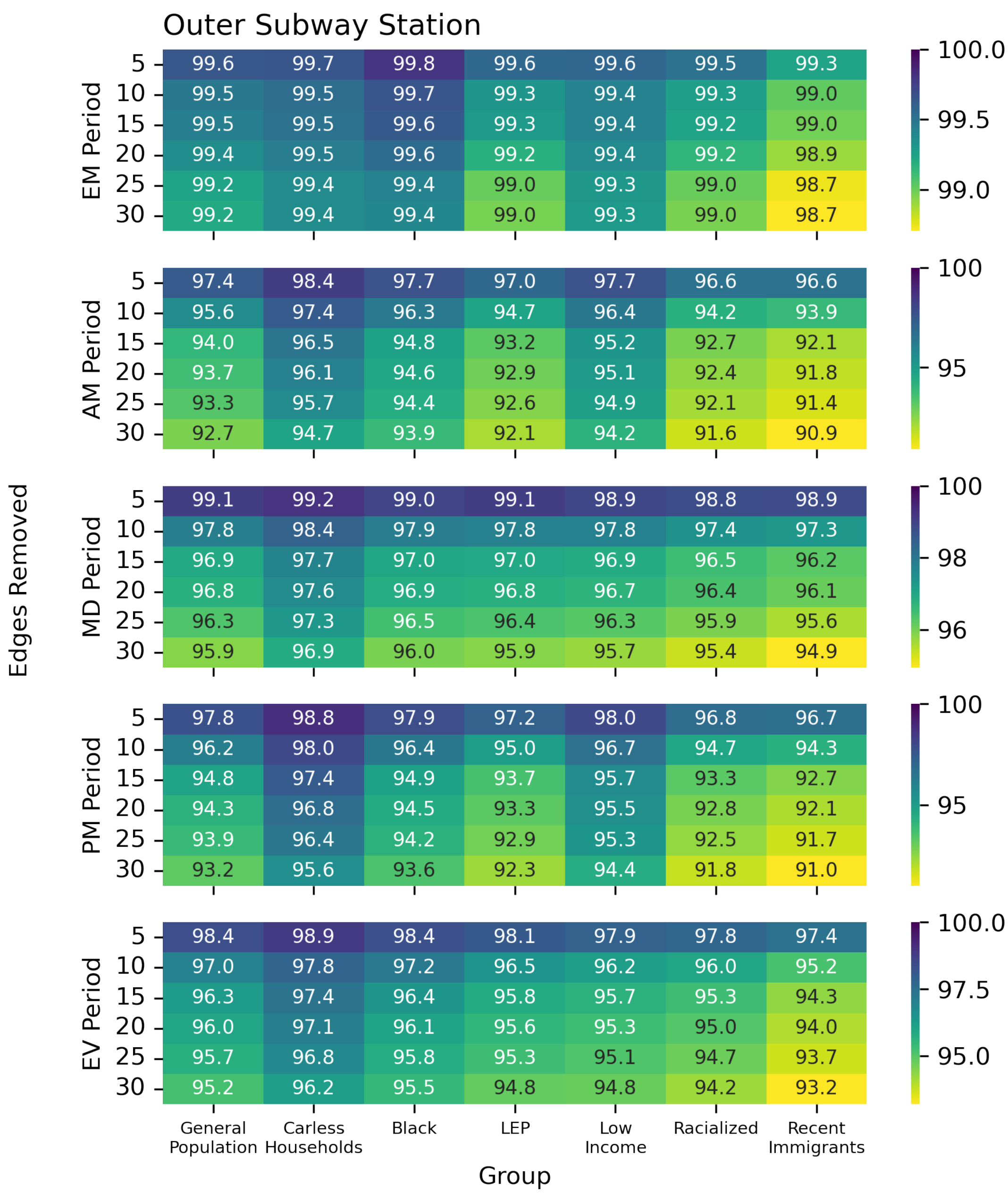

Figure 4.13: PWRGE results after removing edges representing subway segments connecting outer subway stations subway stations.

4.5 Conclusion

In general, the PWBC and PWRGE share many similarities. Both measures determined that Carless Households and the General Population were the most distributed, while the other equity seeking groups were generally more concentrated onto specific stations, route segments, or stations, and would suffer more delays as disruptions continue to occur randomly on the network. This would indicate that during a transit disruption, whether it is random or affects the most critical stations, route segments, or stations, most equity seeking groups would face slightly more disruption impacts than the general population in a hypothetical disruption.

Both measures also show that different population groups had differing levels of criticality for different modes. The bus network, including both frequent and infrequent routes, was most critical for equity seeking groups, excluding Carless Households, while the streetcar network more critical for LEP riders, Carless Households, and to some extent, the General Population. The two measures disagree on the level of criticality the subway network has for different population groups. The PWBC analysis indicated that the subway network has higher importance and criticality for Carless Households and Low-Income commuters, while the PWRGE analysis indicated that Recent Immigrants (and the General Population for inner subway stations) would face higher delays if disruption occurred on the subway network. But the two analyses confirm that disruption mitigation efforts should take into consideration the mode if the goal is to implement transportation justice and transportation equity. While decreasing the importance of individual stations on the subway network may benefit the General Population the most, it would not have as big of an impact on equity seeking groups as ensuring the suburban bus network is unaffected by significant delays.

Chapter 5

Redundancy of Transit Service in the TTC Network

5.1 Introduction

Chapter 5 focuses on three graph measures that analyze redundancy of transit service. In the event of a disruption, areas with redundant transit routes and options would have a less severe impact on groups living in those areas, as they could still reach their destinations, albeit with some delay. Two measures using the route-map graphs were used: connectivity and average neighbouring node degree. Additionally, one measure using the time-expanded graph was used: the additional path factor measure. For the two route-map graph measures, the associated measures were weighted using the equation shown in Equation 5-1. The results for the origin and the destination were averaged, and then weighted by the proportion of all trips making a trip between those specific origins and destinations.

$$M_{weighted} = \Sigma_{od}[\frac{x_{od}}{x} * \frac{M_o + M_d}{2}] \tag{5.1}$$

Where

- $M_{weighted}$ = the population weighted measure of interest, for a specific population group and period
- M_o = measure for the origin ward, for a specific population group and period
- M_d = measure for the destination ward, for a specific population group and period
- x_{od} = = total trips made from origin ward o, to destination ward d, for a specific population group and period

Period	**Black**	**Low Income**	**LEP**	**General Population**	**Carless Households**	**Racialized**	**Recent Immigrants**
EM	7.1	8.0	7.8	7.7	7.6	7.5	7.8
AM	79.8	82.7	83.2	84.6	85.4	85.7	87.2
MD	53.0	54.8	53.5	54.8	55.7	55.2	56.3
PM	78.1	79.6	79.8	81.9	83.0	82.7	83.7
EV	48.0	50.6	49.2	50.2	51.2	50.7	51.4

Table 5.1: Connectivity experienced by each population group, disaggregated by period.

- x = total trips made by a population group and period

5.2 Connectivity Analysis

Connectivity results are displayed in Table 5.1, while Figure 5.1 shows results relative to the population group with the highest connectivity results for each period. Figure 5.2 shows the connectivity score for each ward.

Lower connectivity scores would indicate that the origin and destination of the trips were in wards with a low connectivity score. Low connectivity scores would indicate there were less available options to move around the network, and less opportunities to transfer with other routes, reducing the amount of route choices and redundancy at the start or end of trips. However, the approach used to determine experienced connectivity does not take into account transfer or path choice opportunity in the middle of a commuter's journey.

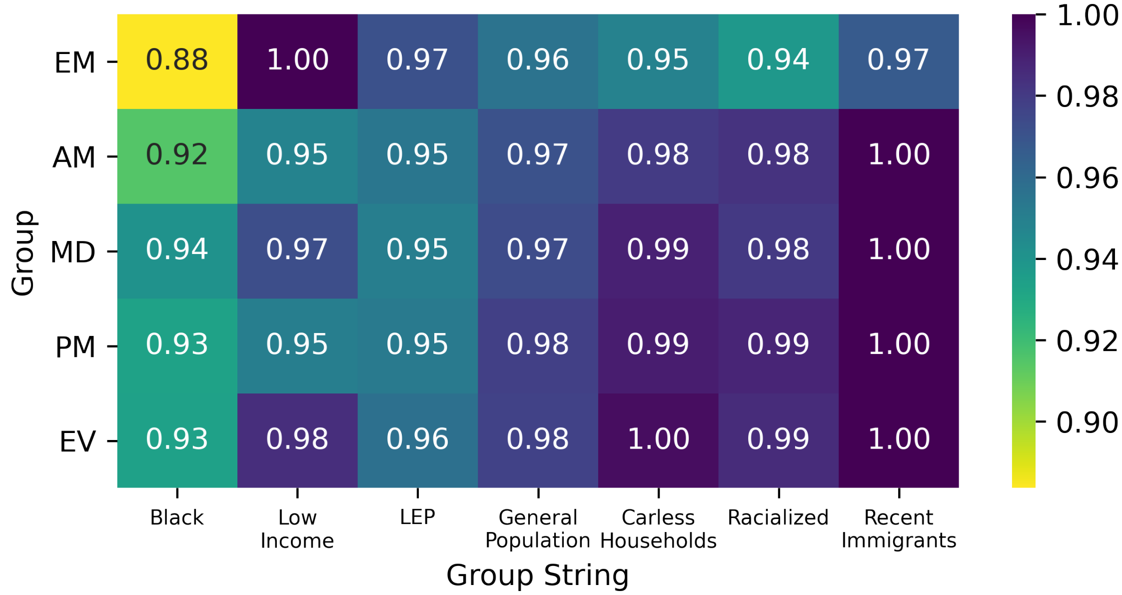

Figure 5.1: Experienced connectivity for each population group, disaggregated by period, relative to the highest connectivity experienced by a population group in a period.

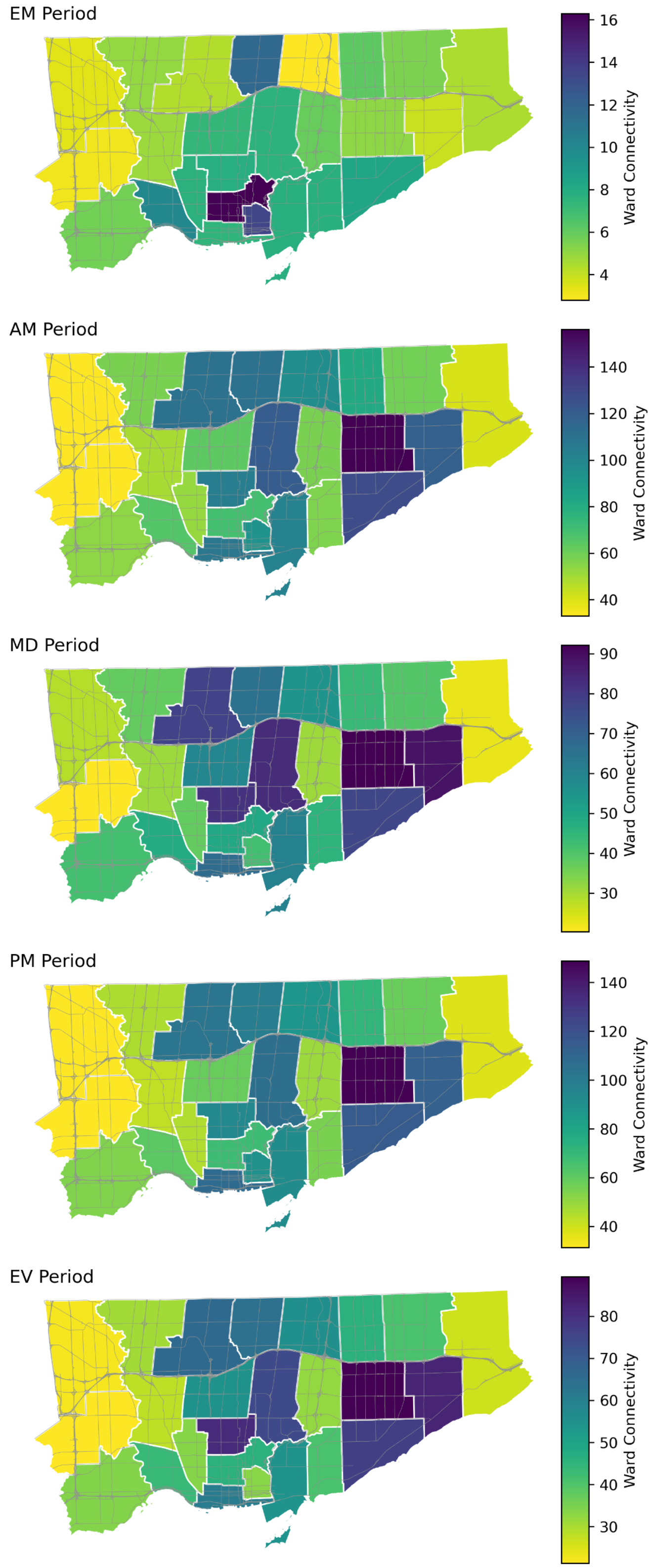

Figure 5.2: Connectivity score for each ward and period. [17]

Results in the early morning were much lower than results in the other periods. This period is especially important for overnight workers returning from their shifts or shift workers travelling to start an early shift. Most equity seeking groups had lower connectivity scores than the General Population for this period. Beyond having lower scores, Black riders and Carless Households would be affected more by the lack of redundancy the low connectivity scores indicated since they have 1.4 times and 1.3 times, respectively, more trips than in the early morning than the General Population.

For the relative results, shown in Figure 5.2, among population groups for the four daylight periods, Black, Low-Income, and LEP riders had noticeably lower redundancy than other groups. Part of this can be explained by the trip patterns of these groups, discussed in Chapter 3. These three groups were also more likely to make trips in areas with low connectivity and transit service, rather than the CBD or the Inner City; LEP riders start or end their trips near northern Scarborough, while Black and Low-Income riders generally start or end their trips in Scarborough or Etobicoke

The raw results in the midday and evening, shown in Table 5.1, were also lower than the two rush hour periods. This would impact Low-Income riders and Carless Households more since they travel more than the General Population in those two periods. Even though their connectivity scores were not much different than the General Population for those two periods, because more of those two groups traveled during the off-peak periods, Low-Income riders and Carless Households would have a greater proportion of riders experiencing lower redundancy.

5.3 Average Neighbouring Node Degree Analysis

Lower ANND experienced by groups would indicate that stops in their origin and destination wards were not connected to nearby stops, or the frequency connecting the stops to nearby stops was low. If service is disrupted on stops with a low ANND, commuters would have less alternatives since nearby stops would also have poor transit service.

Raw results for the average neighbouring node degree (ANND) are displayed in Table 5.2, while the heatmap in Figure 5.3 shows the results relative to the highest value among all population groups in each period. Figure 5.4 displays the ANND score for each ward.

Period	Black	Low Income	LEP	General Population	Carless Households	Racialized	Recent Immigrants
EM	7.1	8.0	7.8	7.7	7.6	7.5	7.8
AM	79.8	82.7	83.2	84.6	85.4	85.7	87.2
MD	53.0	54.8	53.5	54.8	55.7	55.2	56.3
PM	78.1	79.6	79.8	81.9	83.0	82.7	83.7
EV	48.0	50.6	49.2	50.2	51.2	50.7	51.4

Table 5.2: Average neighbouring node degree experienced by each population group, disaggregated by period.

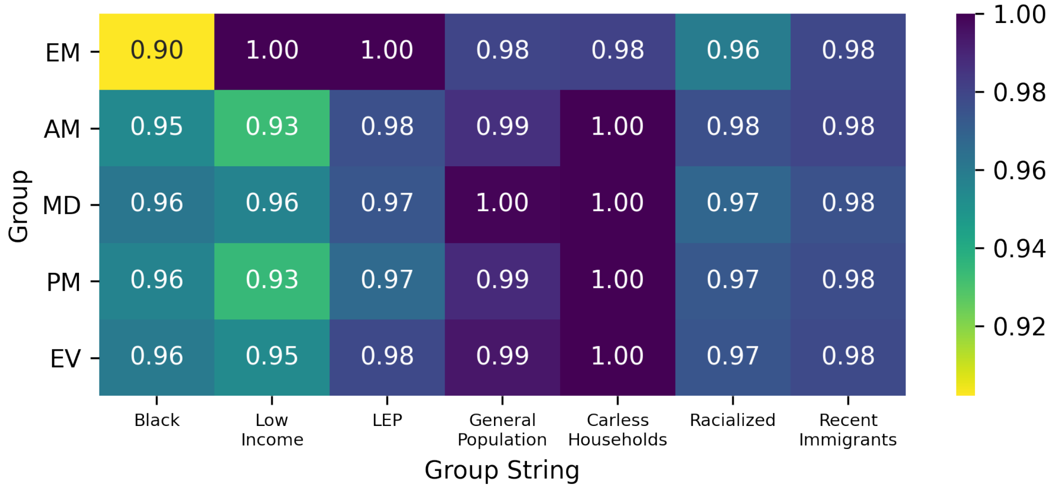

Figure 5.3: Experienced average neighbouring node degree for each population group, disaggregated by period, relative to the highest connectivity experienced by a population group in a periodd.

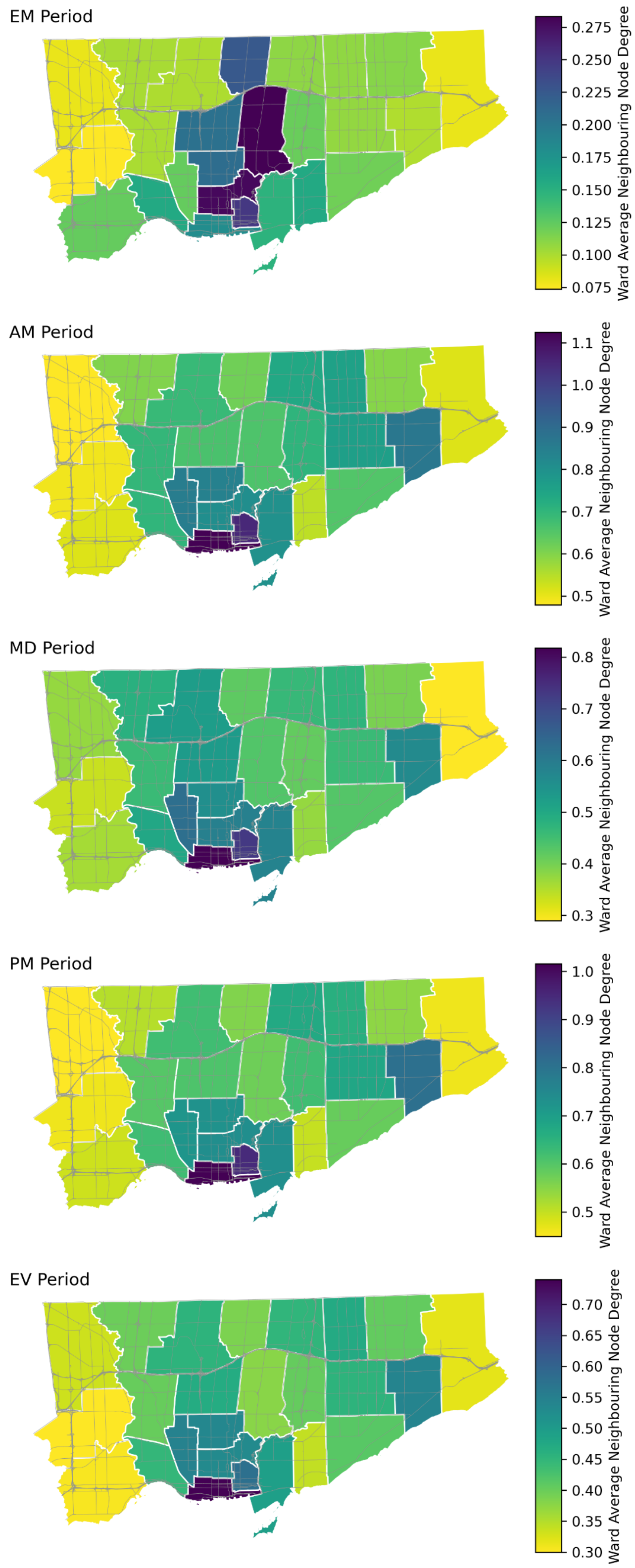

Figure 5.4: Average neighbouring node degree score for each ward and period [17]

Higher ward level ANND were observed in the CBD of Toronto for the four daytime periods, while the three suburban districts had lower scores. For the early morning, higher values were observed in the CBD, Inner Toronto, and North York.

Like the connectivity results, Black riders had lower ANND scores in the early morning. Black riders also had lower scores in the remaining daytime periods, which was due to the fact that more of their trips were made to and from other suburbs. Alongside the fact that a higher proportion of Black riders travel in the early morning, this meant a larger number of Black riders had lower ANND and redundancy.

Similarly, the results for Low-Income riders share much of the same characteristics as the connectivity analysis. They had lower scores than the General Population, especially for the two rush hour periods, and since more of these riders travel during the off-peak periods, a greater proportion of the group has lower ANND scores. Again, this was due to the suburban nature of trips for Low-Income riders.

Since Racialized, LEP, and Recent Immigrant riders had a greater share of non-CBD trips than the General Population, the slightly lower ANND scores for Racialized, LEP, and Recent Immigrant riders was expected.

5.4 Additional Path Factor

Using the process described in Chapter 3.7.3, APFs were calculated for all OD trips made by each population group for each period. Like other time-expanded graph measures used in this study, such as PWBC and PWRGE, the number of paths were calculated for six random departure times within each period and averaged to account for the impact of the departure time.

Figure 5.5 shows that for longer transit trips, such as trips with a length of 60 minutes or greater, the APF would also increase. Longer shortest path travel times (SPTT) tended to have a longer viable path travel time (VPTT), with the additional time buffer added to the SPTT increasing to a maximum of 10 minutes for a SPTT of 120 minutes. Longer SPTT will also cause more path options since there would be an increase in wait/slack time between transfers, which causes more route options and transfers to be viable. To account for APF being higher for longer SPTT and VPTT, the APF results were binned by the SPTT.

Figure 5.6 shows the distribution of travel time by period and group, while Table 5.3 shows the median travel times by period and group. Travel times were longer in the two rush hour periods compared to the two off peak periods, and much longer in the early morning period compared to the other periods. For the four daytime periods, the majority of trips in all groups had a calculated shortest path travel time

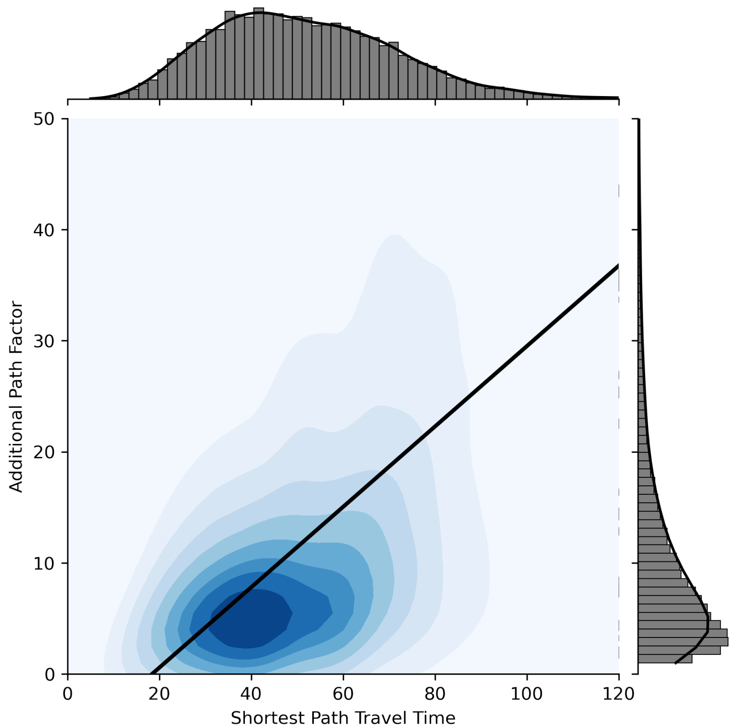

Figure 5.5: Relationship between APF and SPTT, displayed in a KDE scatter plot graph with a regression line.

lower than 60 minutes. For all periods, Carless Households had a lower median travel time than the other population groups. For the remaining equity seeking groups, they had slightly higher median travel times than the General Population. It should be noted that these travel times were calculated from the time-expanded graph, and not from the TTS data.

Period	**Black**	**Low Income**	**LEP**	**General Population**	**Carless Households**	**Racialized**	**Recent Immigrants**
EM	91.5	95.6	93.6	96.1	91.3	86.6	89.7
AM	53.0	54.9	55.1	53.2	55.8	53.2	48.7
MD	45.9	47.5	48.5	46.3	47.7	46.8	42.3
PM	51.3	53.9	54.5	52.2	55.0	53.3	46.2
EV	47.9	49.8	51.0	48.8	50.2	49.0	42.8

Table 5.3: Median travel times by group and period.

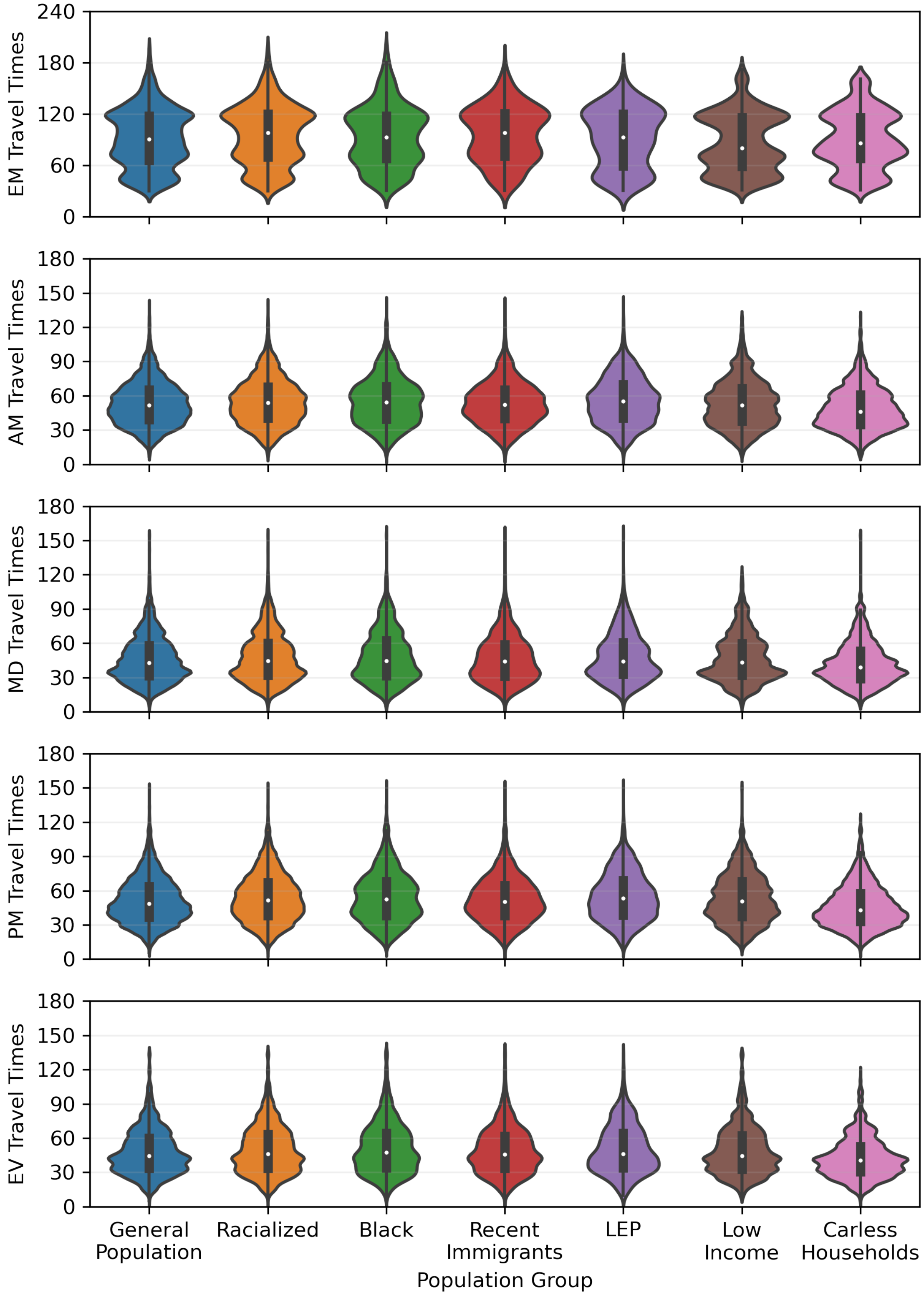

Figure 5.6: Violin plot of the calculated shortest path travel times for each period and group. Median, 25th and 75th percentiles are shown by the small box plot inside the violin. Distributions of travel time are indicated by the shape of the violin.

Figure 5.7 shows APF results for each population group and period, binned by length of SPTT. Alongside transit paths, the results in Figure 5.7 also included paths composed only of walking legs; when walking paths were removed, there was little change between the results that included walking and the results that excluded walking.

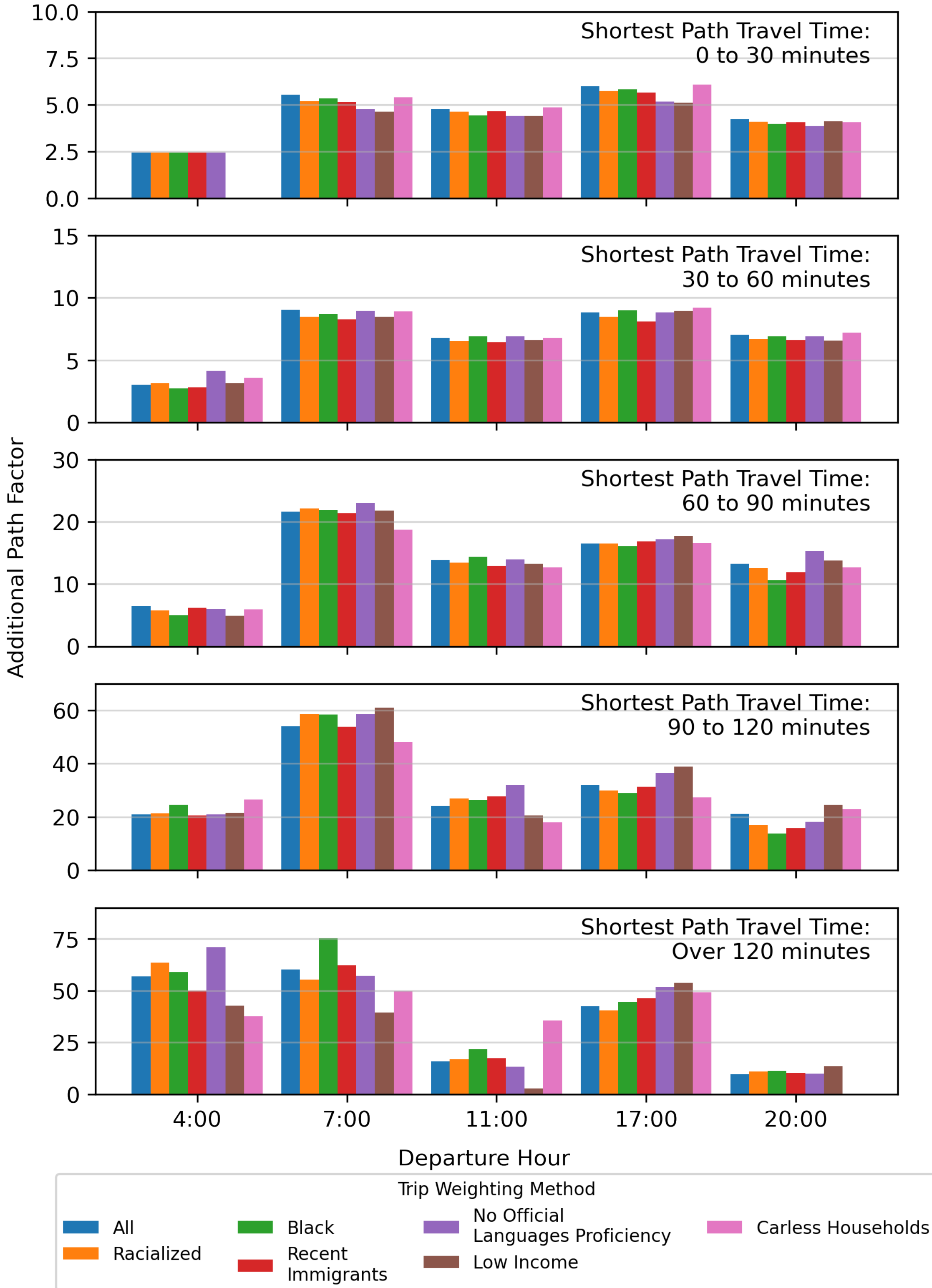

Figure 5.7: APF results for each population group and period, binned by length of SPTT.

APF values for all travel time bines were higher in the midday and evening periods than the morning and afternoon rush hour periods. Since Black and Low-Income riders were more likely to make rides in the off-peak periods than the General Population, more Black and Low-Income riders were exposed to periods with low redundancy. The same was true for the early morning period; this was also compounded by the fact that Black and Low-Income riders were 1.4 and 1.6 times more likely to make trips in the early morning period. Carless Households generally had a similar APF compared to the General Population for trips under 60 minutes. Given that the median Carless Household transit trip was under 50 minutes for all daytime periods, and their median travel time was lower than the General Population, this suggests that Carless Households may have better overall redundancy than the General Population. Carless Households tend to make more trips in the Inner City and CBD than all other groups; those two districts had better transit service than the three suburban districts of Toronto. This would explain the reasoning for the better redundancy this group experiences.

For the rest of the equity seeking groups in the daytime periods, with some exceptions, the general trend was that longer paths resulted in higher APF values than the General Population, while shorter paths would have lower APF values than the General Population. Since equity seeking groups make more intra-suburb trips than the General Population (for example, trips starting and ending in Scarborough), and the suburban districts had lower redundancy than the Inner City or CBD, this would explain the lower APF values. One possibility for the higher APF values that equity seeking groups experience in longer trips was that their trips involve more waiting time in between legs because of their higher reliance on the bus network. This creates a scenario where many options could be pursued due to the long slack times, whereas a CBD oriented trip would be severely disrupted if the subway was taken offline, since the speed and frequency of the subway means it was the clear best option for riders making CBD trips, and over 15 minutes faster than trips not on the subway. Chapter 4 indicates that most equity seeking groups except for Recent Immigrants rely on the subway network less than the General Population, which supports this theory. Since the majority of trips during the daytime periods were under 60 minutes, this means that most equity seeking groups experience slightly less redundancy than the General Population and Carless Households on an overall basis, when comparing within periods.

5.5 Conclusion

Overall, the General Population and Carless Households had slightly better redundancy than other equity seeking groups, particularly when compared against Black, Low-Income, and LEP commuters. Part of this can be explained by trip patterns; the General Population and Carless Households make more trips in the Inner City and CBD and make more trips in the rush hour period when transit service was better.

Experienced connectivity and experienced ANND are graph measures that produce similar results. Both measures share two weaknesses; the measures were evaluated at the origin and destination while ignoring the redundancy in the middle of the journey, and the measures use computed measures for an entire ward which may ignore the spatial differences inside the ward.

The APF measure somewhat confirms the results shown by the connectivity and ANND results, and additionally demonstrates that for longer trips, most equity seeking groups had better redundancy than the General Population and Carless Households. Since most trips made by transit were under 60 minutes, the general result was that most equity seeking groups for most periods have slightly less viable alternative path options.

Chapter 6

Impact of Delays and Disruption Using Real Time Data

6.1 Introduction

The previous results discussed in Chapters 4 and 5 were computed on graphs built on a 2016 GTFS static feed. The GTFS represents a planned schedule that a transit agency plans to run and does not reflect actual service levels posted. Chapter 4 analyzed which group was more exposed and vulnerable to a hypothetical disruption based on the criticality of stations for each population group. Chapter 5 analyzed the redundancy that different population groups had, and the amount of alternative path options in the event of a hypothetical disruption. Using real time data would demonstrate the effects of real-world disruptions on different population groups and demonstrate how well those groups were able to mitigate the disruption.

As mentioned in Chapter 3, real time data representing 23 days between November 28, 2019, and January 3, 2020, was acquired to build 115 separate graphs, each representing a period-date combination. Data during weekends and holidays was excluded.

In addition, static graphs representing the November 24, 2019, GTFS feed were used as a baseline. For each graph, the population weighted global efficiency was calculated, similar to Chapter 4. Then, the population weighted global efficiency of the real-time graphs were compared against the results from the static November 24th graph to create a series of PWRGE measures. Global efficiency was chosen as it would indicate the additional delay caused by disruptions or routine delays. This would measure the level of delay and disruptions commuters experience, and whether they had redundant options available. The measure was also quick to compute compared to other measures needing the time-expanded graph, which was a critical factor since the measure would need to be used on 120 separate graphs.

6.2 Baseline Population Weighted Global Efficiency Results

The raw PWGE scores were calculated for the baseline graph, representing the November 24th GTFS feed. Ratios between the PWGE scores of each equity seeking group and the PWGE scores of the General Population were calculated and displayed in a heatmap in Figure 6.1.

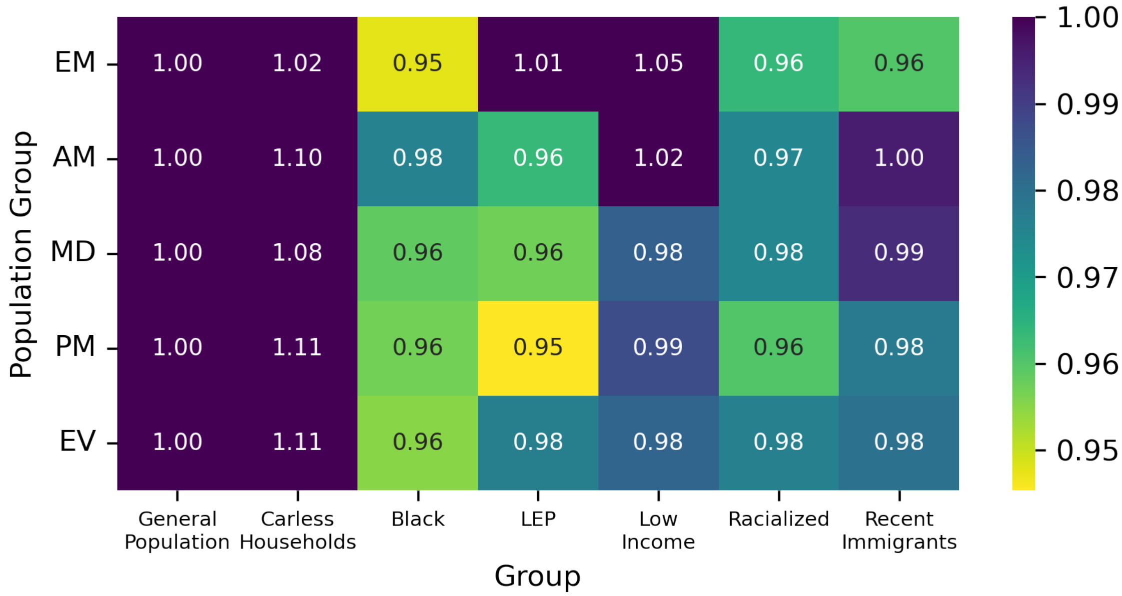

Figure 6.1: Ratios of PWGE scores between each equity seeking group and the General Population, for each period.

Similar trends to the baseline results in Chapter 4 exist. Carless Households had a consistently higher PWGE then the General Population. Recent Immigrants and Low-Income riders had a similar PWGE to the General Population while Black, LEP, and Racialized riders had a lower PWGE than the General Population. For these three groups, they face longer travel times than the General Population if actual transit service matches what was planned in the GTFS. Successive sections will explore if these differences remain during actual operations.

6.3 Summary of Disruptions

After creating the graphs, travel times between all OD pairs in the GTFS were computed for all real time data graphs, and the static graphs. Total travel times for the real time data graphs were then compared against the total travel times for the static graphs. These results are displayed in Figure 6.2.

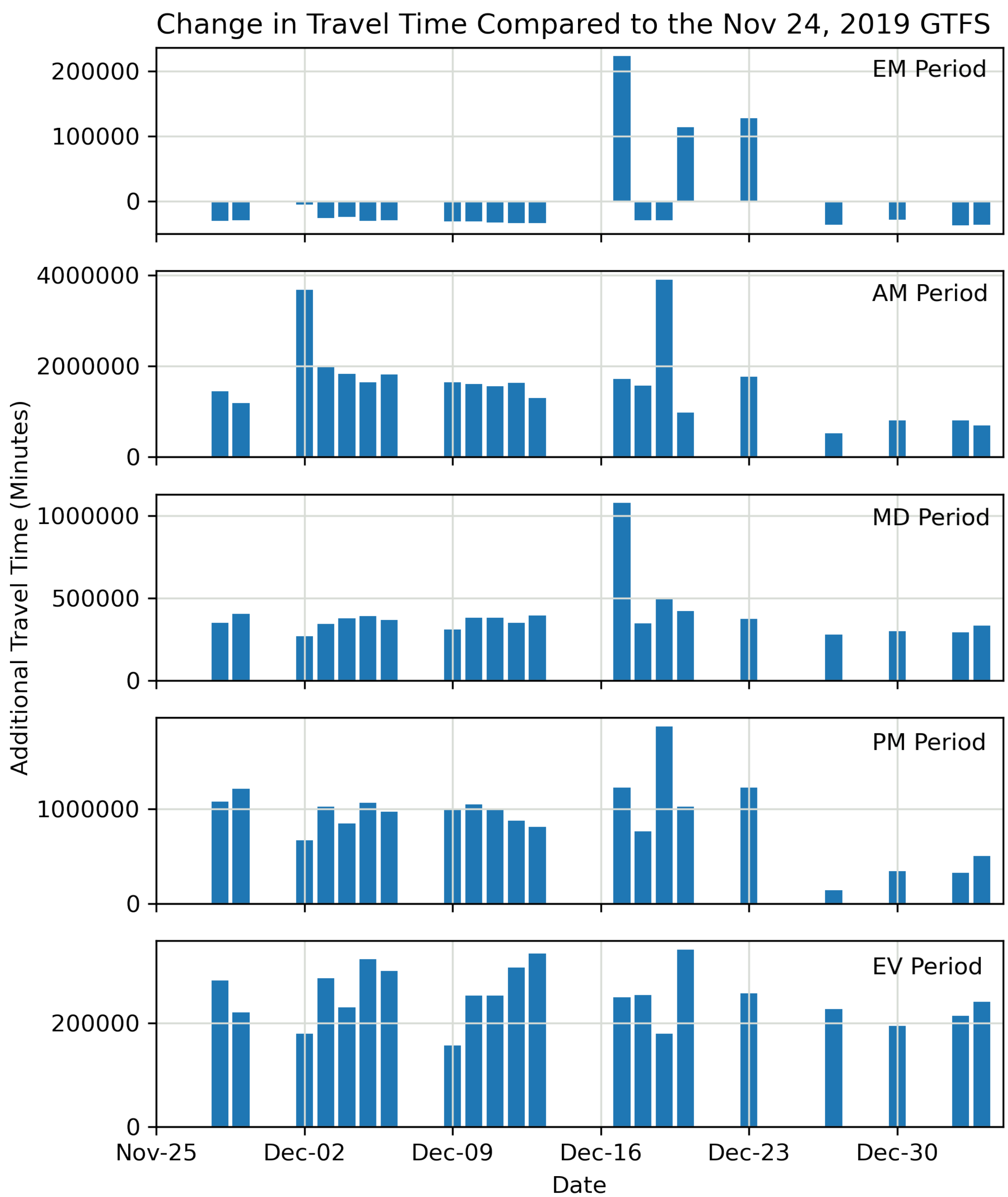

Figure 6.2: Total travel time of trips made by the General Population comparisons between the real time data graph, and the November 24th static graphs, for each period. The y-axis for each subplot is different.

In general, except for the EM period, all dates and periods had excess travel times compared to the baseline static graph. This was usually caused by delays, disruptions involving service shutdowns/diversions, or missing trips. Disruptions did not affect exclusively a single mode; disruptions affected all three modes in the TTC network. Every date and period had a route or multiple routes with some form of delays and disruption. These disruptions were referred to in this project as routine delays or routine disruptions, since they collectively occurred on a frequent basis. However, there were five periods that were identified as having significant disruptions since the excess travel time was much higher than for other dates and periods:

- December 2nd AM Period
- December 17th EM Period and MD Period
- December 19th AM Period and PM Period

Using information from the TTC Service Alerts twitter account [84], the cause of the excess total travel time for these periods was due to winter weather, such as snow and ice, affecting the majority of surface routes and subway lines.

6.4 Real Time Data PWRGE Results

Results are displayed in Figure 6.3. The figure contains six charts, each chart comparing the PWRGE between the General Population and an equity seeking group. All five periods are represented on the chart, and each period is represented by a different coloured point. The PWRGE result for the equity seeking group is represented on the y-axis, while the PWRGE result for the General Population is represented on the x-axis. Each point on the chart represents the PWRGE result for a different day and period combination; if the point is above the identity line (shown on all the charts), then the equity seeking group suffered less delays and disruption relative to the General Population compared to the baseline GTFS feed. This means the total travel time for that day/period combination was closer to the GTFS than the corresponding total travel time for the General Population. Disruptions can also be identified on the figure; outlier points that were not grouped with other points in the same period represent disruptions. For example, the two orange points with a PWRGE of roughly 80% in all six charts represent the December 2nd and December 19th disruptions in the morning rush hour.

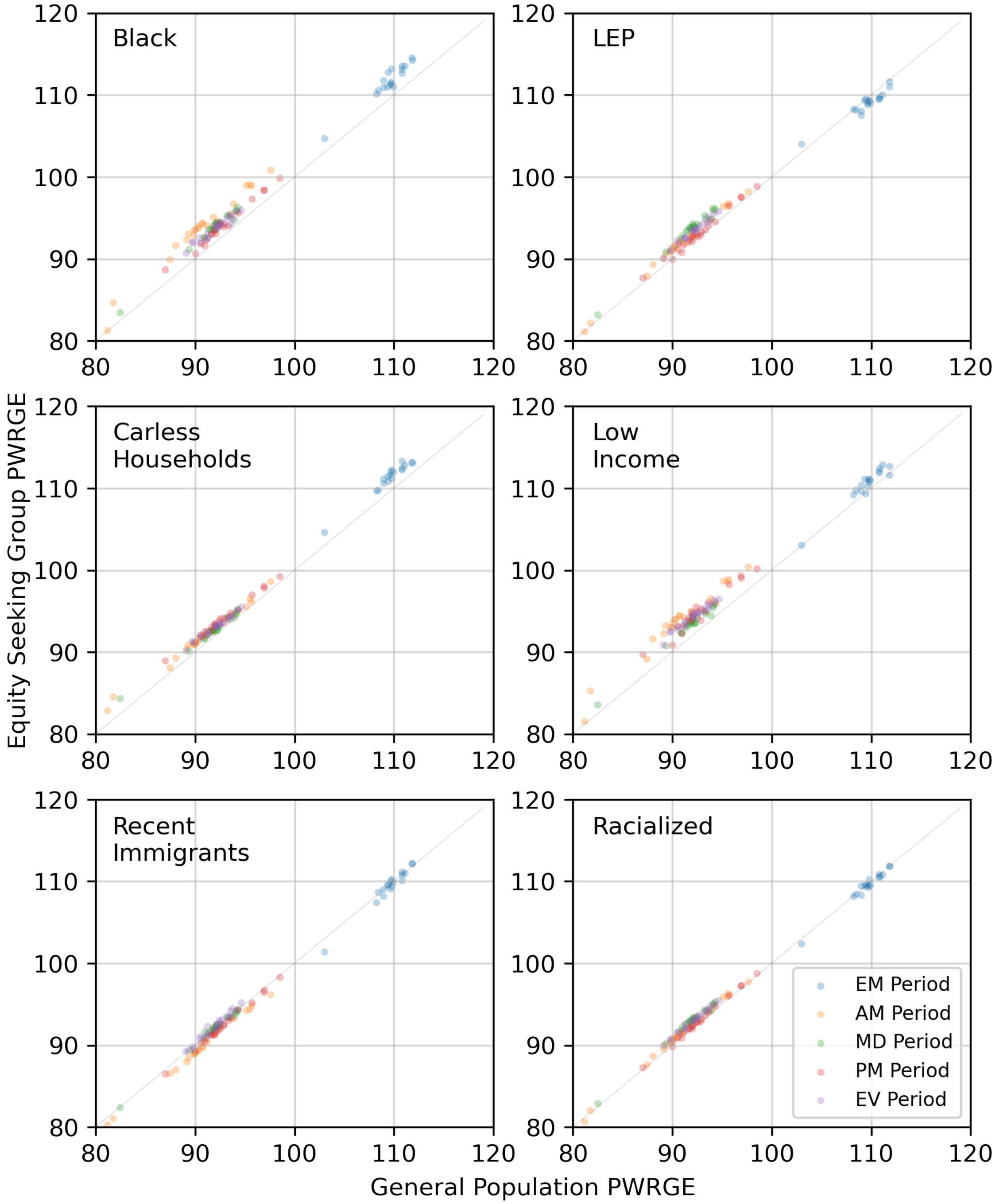

Figure 6.3: Comparison of PWRGE between equity seeking groups and the General Population. Additional figures for each period are shown in the Appendix C.

Mirroring the information from Figure 6.1, the PWRGE for the early morning period was above 100%. Since Black, Low-Income, and Carless Households were more likely to make trips in the early morning, they benefit more than the General

Population from the faster than expected travel time. The high PWRGE results may be due to the extra buffer being added to the bus schedules, low traffic in the overnight periods, and low demand for boarding and alightings on the buses.

Besides the higher PWRGE results in the early morning, the results for all periods and groups follow a similar trend. For days and periods that experience routine delays, with an equity seeking weighted RGE and General Population RGE of around 90-95%, most equity seeking groups had a similar or lower travel time increase from delays compared to the General Population. For dates and periods that experience major transit disruption, with PWRGE values closer to 80%, the same conclusion can be drawn; equity seeking groups had a lower PWRGE drop and can reach their destinations with a less relative drop in their travel times during major disruptions than the General Population. Black and Low-Income riders performed consistently better than the General Population, while Recent Immigrants performed slightly less than the General Population; Racialized, Carless Households and LEP riders performed roughly similar or slightly better compared to the General Population, but not to the same degree as Black and Low-Income riders. For each date and period combination, ratios were calculated between the PWGE scores of each equity seeking group and the General Population. The ratios are displayed in a violin plot in Figure 6.4.

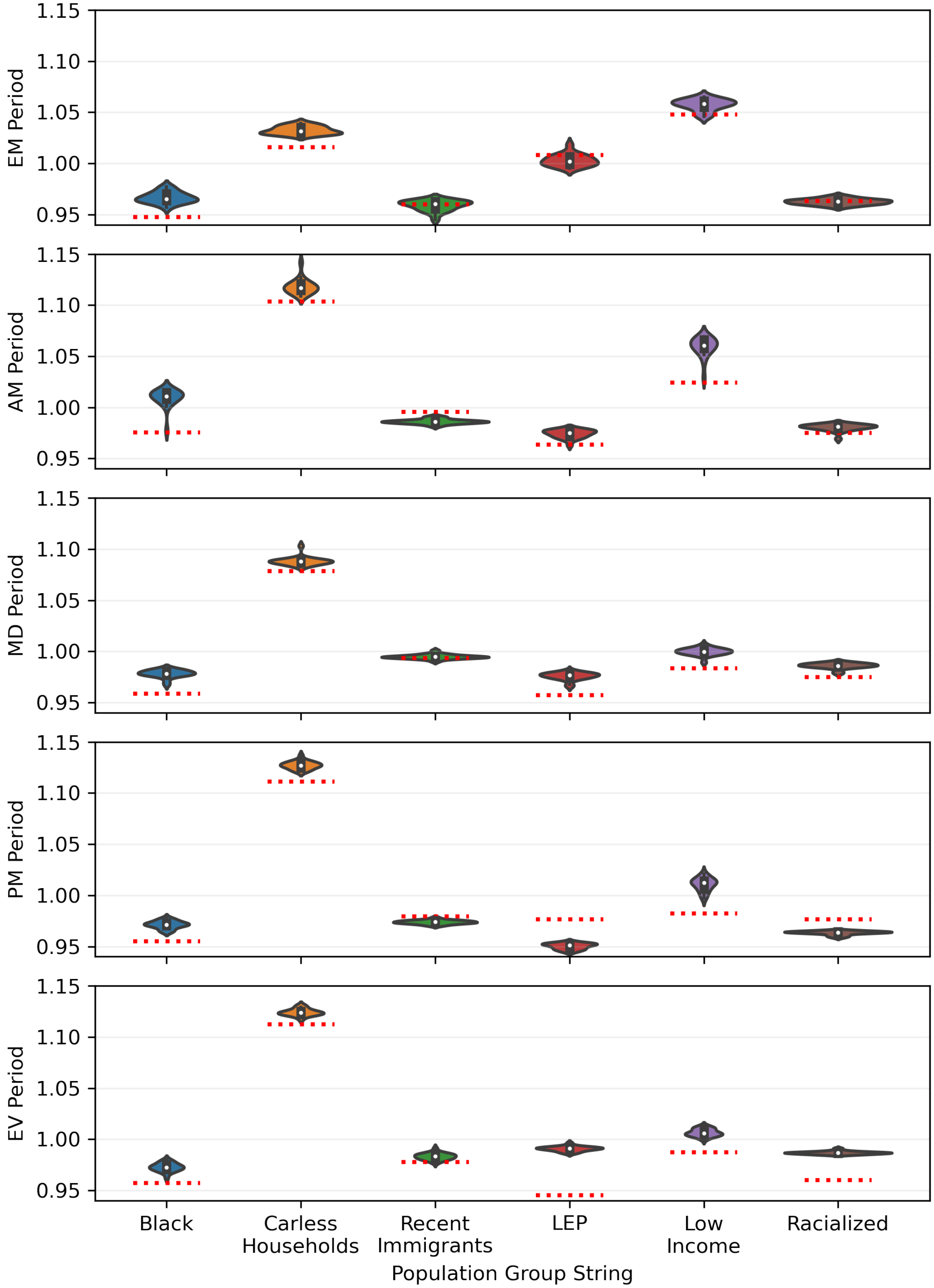

Figure 6.4: Violin plot of the ratios between the PWGE scores of each equity seeking group and the General Population. Median ratios are indicated by the white point inside the violin, while the shape of the violin indicates the distribution of the ratios.

Compared to the baseline results, the median ratios for Black and Low-Income riders increased, showing the PWGE for these equity seeking groups had either matched the PWGE scores of the General Population, or surpassed it. Carless Households, Racialized and LEP riders experienced similar trends, but to a lesser degree. For Recent Immigrant riders, the ratios were slightly lower compared to the baseline results. This figure matches the results in Figure 6.3; for equity seeking groups that experienced a lower relative PWRGE drop, the ratios increased, and vice versa. For the AM period, this means Black and Low-Income riders usually have lower travel times compared to the General Population, since the ratios were above one, which would not be the case if transit service matched the planned schedule. This conclusion may seem counterintuitive, but some explanation can be drawn from closer inspection of the travel patterns. Black and Low-Income riders were more likely to make intra suburb and inter suburb trips than the General Population. They were also more likely to rely more on the bus network than the General Population. These trends hold true for Racialized, Carless Households, and LEP riders, except they make less non-CBD trips than Black and Low-Income riders and the bus network had a lower criticality for those groups than for Black and Low-Income riders. While all modes experience some sort of delay and disruption (especially for weather related delays seen in the disruption on December 2nd and 19th), buses were less likely to completely shut down service in the event of a disruption compared to rail modes. Research from 2015 suggested there were on average 2.5 and 0.4 incidents per day on the streetcar and subway networks respectively, and each streetcar and subway incident caused an average of 86.7 and 86.0 minutes of delay, respectively [34].

The importance analysis in Chapter 4 implicitly assumes any disruption to a route or station will lead to a shutdown of that route segment; however this is not as true for bus routes compared to rail routes. These groups were also less likely to travel to the CBD. Disruptions on the subway and streetcar network, which feeds a large portion of riders to the CBD and Inner City, would be less impactful to Black and Low-Income riders compared to the General Population. Black and Low-Income riders had a similar level of criticality for the outer subway network compared General Population, however given their travel patterns, many riders could avoid the subway network in a delay since they were paths that can avoid using suburban subway stations aside from transferring from one bus to another. In contrast, Chapter 4 has shown that Recent Immigrants had a similar PWRGE decrease compared to the General Population for the inner subway network, and the fastest PWRGE drawdown for the outer subway network, showing their reliance. Out of all the equity seeking groups, they also had the highest number of trips originating or ending in the CBD.

These facts demonstrate why they may have a slightly higher disruption than the General Population.

6.5 Conclusion

Building time-expanded graphs using real time data can be a viable way to analyze real world conditions of transit operations. While the datasets may have missed some transit trips and did not completely model all transit service during the 23 dates, it allowed some clear conclusions to be drawn. The early morning period consistently allowed commuters to arrive at their destination faster than planned in the GTFS schedule, while for other periods, there was an up to 15% decline in the population weighted global efficiency compared to the baseline network.

Comparing PWRGE results for equity seeking groups to the General Population allowed an analysis of the relative decline of service during both routine delays and transit disruptions to take place. In real world situations during the winter of 2019, equity seeking groups generally had a roughly similar level of delay during routine delays and disruptions compared to the General Population. This finding was interesting given that prior analysis involving the static GTFS feed showed that equity seeking groups would be slightly more exposed to disruptions and had slightly less redundancy. However, the real time analysis can be explained by analyzing the travel patterns of equity seeking groups and matches prior studies that compared the on time performance of bus routes between minority and non-minority populations in Toronto [67]. While it is true that equity seeking groups, as shown in Chapter 4, were slightly more concentrated onto specific routes and nodes, in reality those routes and nodes may be less likely to fail. This analysis did not determine if baseline travel times were equitable as this analysis was determining if the change in travel times during delays and disruptions were equitable. It's possible that travel times after the delay or disruption may still not be equitable, even if the increase in travel times due to delay or disruption was equal or equitable. Past accessibility research on the planned TTC schedule has shown that the system was equitable, or at least equal for equity seeking groups [1, 37, 39].

This phase of the analysis did not include run as directed (RAD) buses. During normal operation RAD buses provided supplemental service to bus and streetcar routes, but during major subway disruptions, RAD buses were diverted from bus and streetcar routes, and act as a bus bridge. RAD buses acting as subway replacement buses were not found in either of the two data sources used to create the graphs, and there's some evidence from transit advocates that RAD buses were not shown in the NextBus API when they were supplementing service to regular bus routes during normal operation (Steve Munro). This may have made the pathfinding and travel times during subway shutdowns more optimistic since it ignored the possibility that bus bridging services were available.

Another limitation was that this implementation of the time-expanded graph had no capacity constraints. If capacity constraints on transit vehicles were implemented, path itineraries may change. During a disruption or delay, vehicles can quickly reach capacity and prevent riders from boarding the vehicle despite vehicles arriving at the stop. This may indicate the level of delays predicted by this graph was optimistic and travel times were longer than calculated. Future work could investigate if implementing capacity constraints on the graph model would lead to a different conclusion.

Chapter 7

Conclusion

7.1 Research Novelty

Graph/network theory, disruption, vulnerability, importance, and redundancy are not new concepts in public transit research. However, prior studies around those concepts focused primarily on rail networks, service in the peak periods, and ignored how headways and timetables may affect the research questions under investigation. While some studies have integrated frequency into an L-space/route map graph, and others have implemented a more limited version of a time-expanded graph, this project has one of the largest implementations of a time-expanded graph to analyze the multi-modal transit network in Toronto. In addition, this project adopted the approach of using multiple and randomly selected departure times from accessibility research to a disruption and reliability study to ensure results are not affected by low frequency routes.

With the time-expanded graph, many graph measures could not be used as traditionally formulated. The project revised the betweenness centrality and the global efficiency measures so they could be applied to a time-expanded graph. The study also introduced the additional path factor measure to consider redundancy in a time expanded graph.

The importance of equity and justice has risen in transportation discourse over time. To perform a proper social equity analysis, OD trip flows should be analyzed. For a social equity analysis on disruption, a population weight was applied to all graph measures to determine if different population groups have different levels of exposure, vulnerability, and resilience to a disruption.

Lastly, historical subway, bus, and streetcar tracking data was used to build a graph that represents the state of the transit network on specific days in 2019. Planned schedules found in GTFS feeds do not capture the routine delays, missing trips, and service shutdowns that are experienced by riders on a daily basis. Creating and analyzing a time-expanded graph based on historical real time data is another contribution this project has accomplished, allowing for a true assessment of the level of delays and disruption felt among different population groups.

7.2 Key Findings

While results varied among equity groups and periods, in most cases the General Population and Carless Households were slightly less concentrated onto the busiest intersections and stations, and therefore less vulnerable to disruptions than equity seeking groups. The random removal PWRGE analysis showed that Carless Households and Low-Income riders were less vulnerable to disruptions, while the General Population had a similar or slightly higher global efficiency score than Black, Racialized, LEP, and recent immigrant riders.

Both the betweenness centrality and global efficiency analysis confirmed that the relative importance and criticality of specific transit modes varied between different population groups. Most equity groups relied much more on the bus (both frequent and non-frequent routes) network, with Black riders the most reliant. The streetcar network was more important and critical for Carless Households, LEP riders, and the General Population. The two measures generally agreed that the differences in criticality of the subway network among population groups were small, but the two measures differed in the relative order of importance for the various population groups.

When looking at redundancy, resiliency, and the ability of different population groups to recover from disruptions, Black, LEP, and Low-Income riders generally have lower connectivity and average neighbouring node degree scores. Both these measures indicated the level of redundant service in a ward. Except for the early morning period, the General Population and Carless Households have a higher number of alternative paths than other equity seeking groups for trips under 60 minutes, while for trips longer than 90 minutes, most equity seeking groups have a higher number of alternative paths than the General Population. Given that the majority of trips are under 60 minutes, most equity seeking groups have slightly fewer path options compared to the General Population.

However, when looking at real time data during the winter of 2019, the results suggested that for both routine delays and major transit disruption, most equity seeking groups have similar or better PWRGE than the General Population. This indicates that for a real-world case, most equity seeking groups were able to travel without suffering the same levels of delays compared to the General Population. This is true for all dates and periods analyzed. Compared to the analysis in Chapter 4, these disruptions were not random, and rail disruptions were more likely to cause a shutdown in service, which explained the discrepancy between Chapters 4 and 6.

The results in Chapters 4, 5, & 6 could generally be explained by the trip patterns of the different population groups; equity seeking groups make less trips originating in the CBD and Inner City Toronto compared to the General Population and make more intra and inter-suburb trips.

7.3 Policy Implications

The results show that equity seeking groups have somewhat lower redundancy and slightly higher vulnerability to transit disruptions than the General Population. While the analysis based off the GTFS data show that certain disruption-related graph measures were similar or slightly better than the General Population, this goes against the spirit of equity. Prior chapters have cited the importance of transportation equity; population groups that have the greatest need and the greatest historical barriers should get better transportation service in order to access the same level of opportunities as the rest of the population. Therefore, the overall state of the network is not equitable in relation to disruptions; the system could be designed in a way to reduce the likelihood of disruptions on equity seeking groups, and more redundancy is needed in the event of a disruption. While Toronto has a strong grid network of trunk routes, the bus network strongly incentivizes passengers to choose trunk suburban bus routes, and the rail network is geared to those travelling downtown. To distribute passengers away from busy routes, stations or hubs, service could be bolstered on lesser used bus routes, such as those that do not follow the arterial roads, and new routes could be created that relieve the current trunk routes and end at lesser used subway stations or transit hubs. This would provide more route choice to riders, which is needed in the event their primary choice becomes unusable due to disruption and delay.

Chapter 4 highlights the differing levels of importance among modes. While capital investment in recent years has been concentrated on the subway network, and to a lesser extent revitalizing the streetcar network, it is important to recognize that most equity seeking riders rely on the suburban bus network, especially since the origins and destinations of their trips are often in areas that will never receive rail service in the foreseeable future. While Toronto's bus ridership has not suffered to the same degree as other north American cities, equity seeking riders could stand to benefit by increased investment into making the bus network more reliable and less vulnerable to disruption. While these measures will not change the results of the analysis since much of the analysis was based off static GTFS data, bus priority measures, dedicated right of ways, and new bus technology (particularly electrification to reduce the impact of snow on bus ridership) are ways to make the bus network more robust.

In addition, this project has also highlighted the difference that real time data has with static GTFS data. Many equity studies, performed by both transit agencies and researchers, show changes in service hours or accessibility based off static GTFS or schedule data. However, real time results, either from a GTFS-RT or NextBus feed, could indicate a different equity outcome. This conclusion does not just apply to transit equity or disruption research, other realms of transit operations and service planning could stand to benefit from more use of real time data that allows a more accurate view of the ground truth.

7.4 Limitations and Further Research

While advances in computing have allowed more detailed and accurate graphs to be used to represent transit networks, memory limitations still limit the number of paths that could be generated, the size of graphs, and the length of paths. It is hopeful that advances in computing in the future will allow for graphs that completely integrate the road and sidewalk network, allow all stops to be represented, and allow departure and arrival times to be represented to the nearest second. Currently, transfers were only permitted if two routes served the same intersection or station, but some passengers may be willing to walk to nearby intersections to make transfers and representing the road and sidewalk network would allow this behavior to be represented.

In addition, this implementation of the time-expanded graph did not consider the impact vehicular capacity had on pathfinding. When buses or trains reach capacity, riders would be forced to either wait for the following train or bus or choose an alternative route. Without capacity constraints, the graph model could assign too many riders onto certain routes and return an optimistic travel time.

While the time-expanded graph was unique in incorporating the role of headways, the graph ignored other factors that impact passenger trip choice. This study implicitly assumes travel time is the most important factor in choosing a path; however, other factors such as the number and type of transfers, station design, vehicle mode, public realm of the street, and others, all play a role in how riders choose their paths. To improve the accuracy of path finding measures, other factors can be incorporated into the graph to provide a more accurate pathfinding model.

The TTC allocates a number of buses on standby to provide additional service on bus routes in regular operation but can be called to act as a bus bridge in the event the subway network is shutdown or disrupted. The NextBus API does not appear to include information on these buses, either in regular operation or during bus bridging, so the real time ignored the fact that passengers could still use bus replacement shuttles during major disruptions.

Lastly, the biggest limitation of this project is the estimation of trips for each population group. More advanced probabilistic methods could be used to estimate trips from groups not included in the TTS. In addition, the data used in the project is not at a sufficiently disaggregate scale. This does not allow for the estimation of trips made by riders belonging to more than one population group, such as an estimation of trips made by Recent Immigrants who also belong to Carless Households. Additional work could be done on creating more accurate trip records for each population group if there was access to more disaggregate census data rather than the publicly available census profiles used in this project. However, the easiest solution to correct the demographic data limitation would be to incorporate more questions about demographics in the TTS itself, rather than attempting data fusion between the TTS and the census.

Appendix A

Additional Figures For Chapter 3

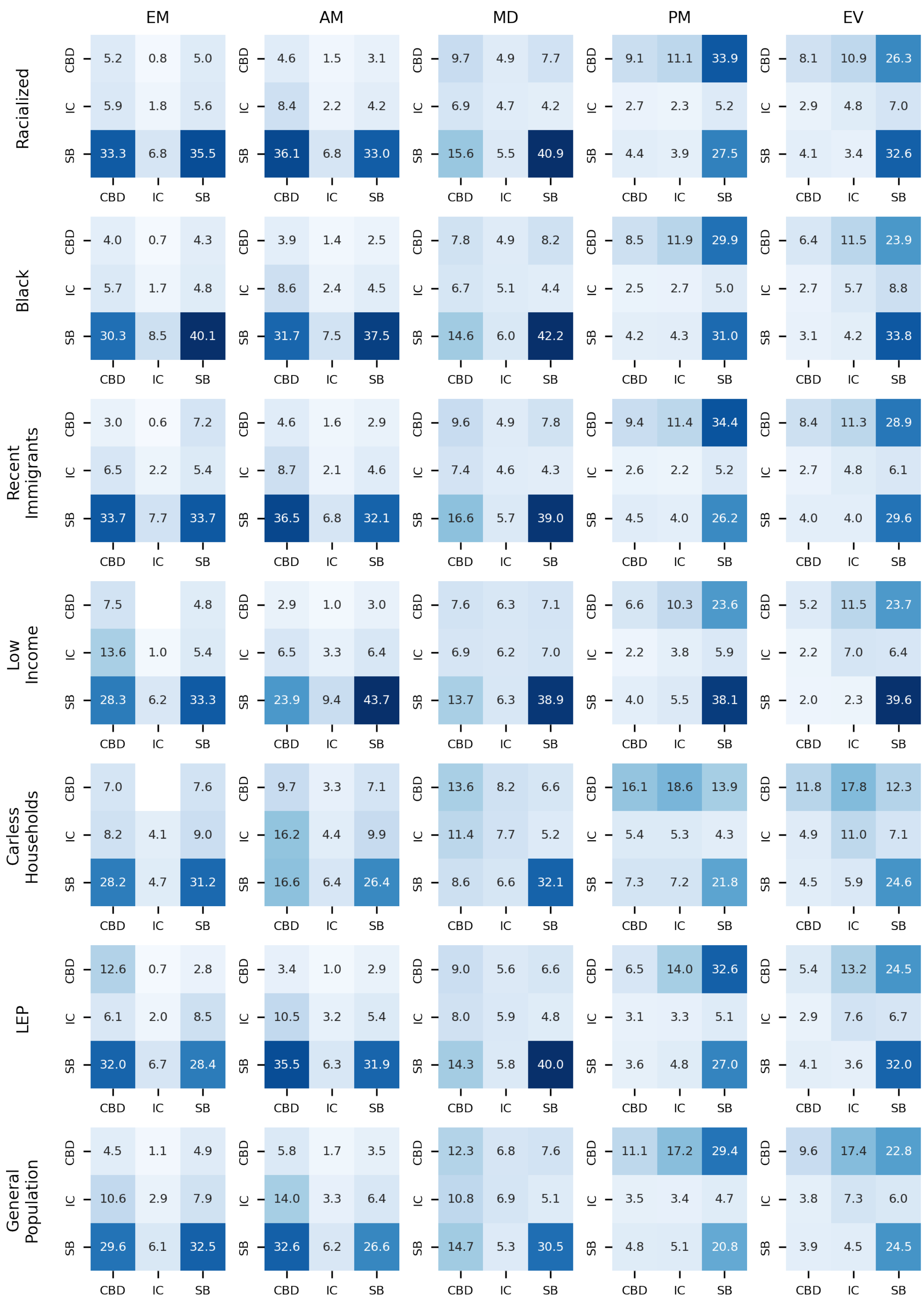

Figure A.1: Simplified trip flow heatmaps for all periods. CBD represents Central Business District, IC represents Inner City Toronto, and SB represents Suburbs (Includes Etobicoke, North York, and Scarborough)

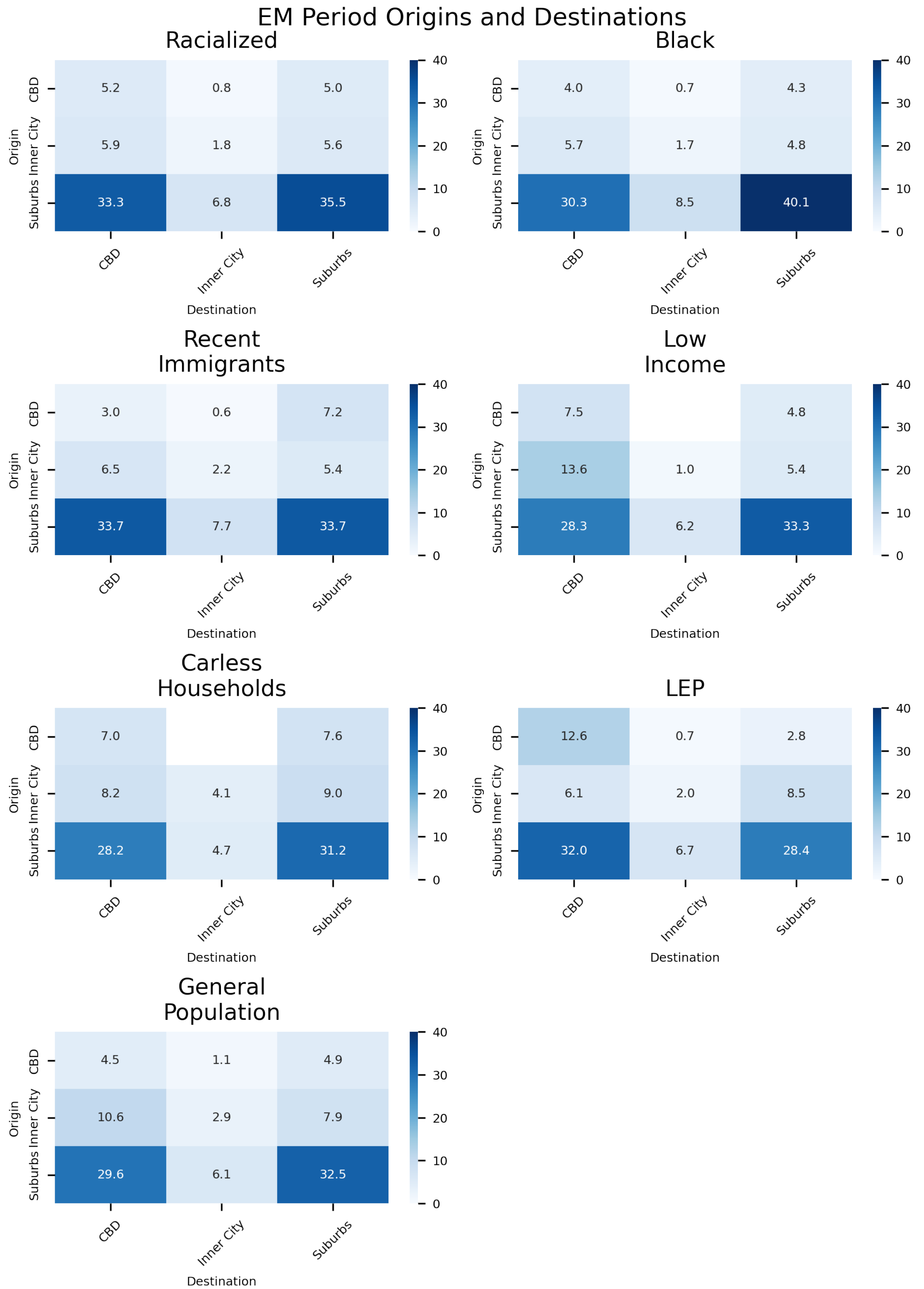

Figure A.2: Simplified trip flow heatmaps for the EM period.

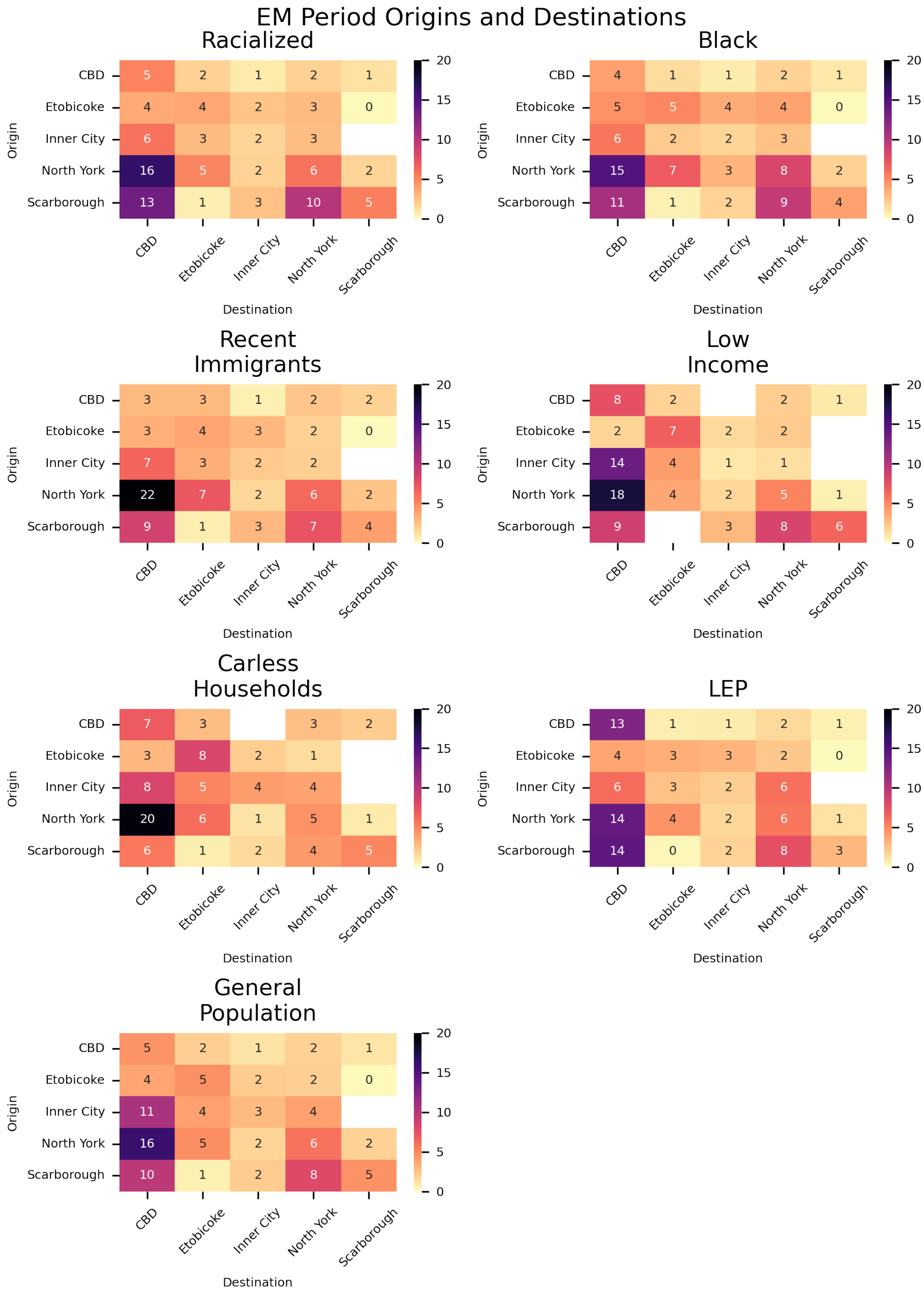

Figure A.3: Trip flow heatmaps by district for the EM period.

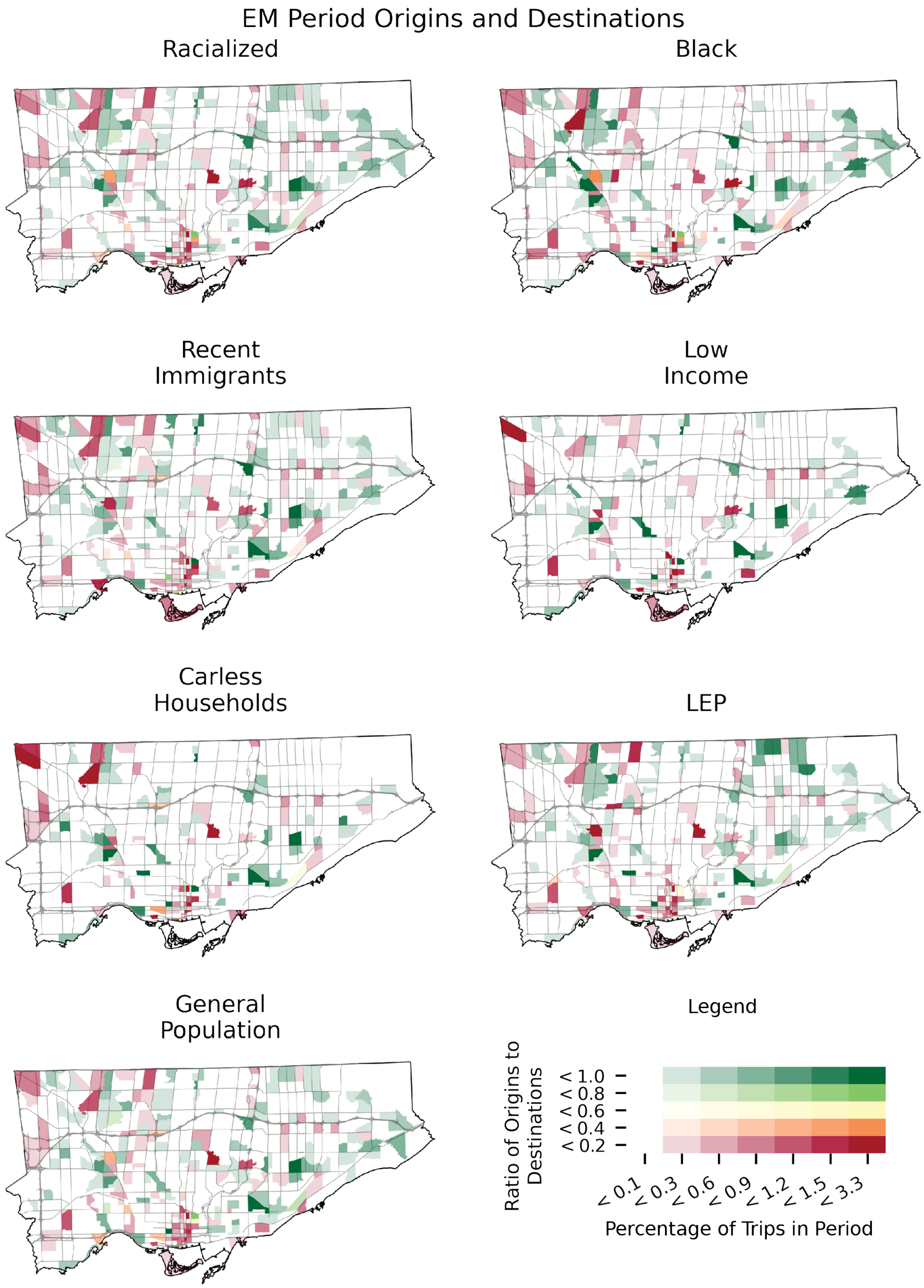

Figure A.4: Bivariate OD trip flows by TAZ for the EM period.

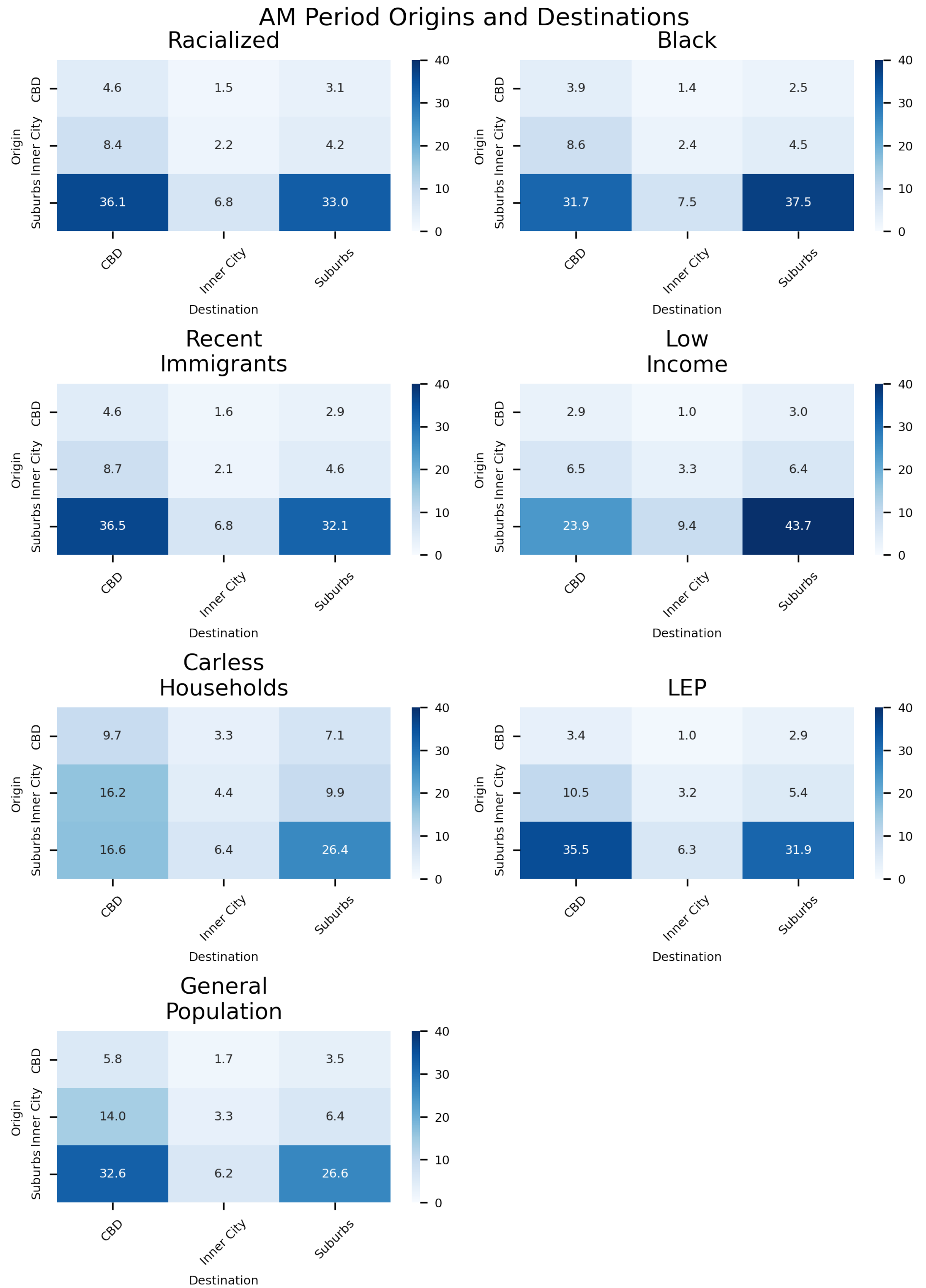

Figure A.5: Simplified trip flow heatmaps for the AM period.

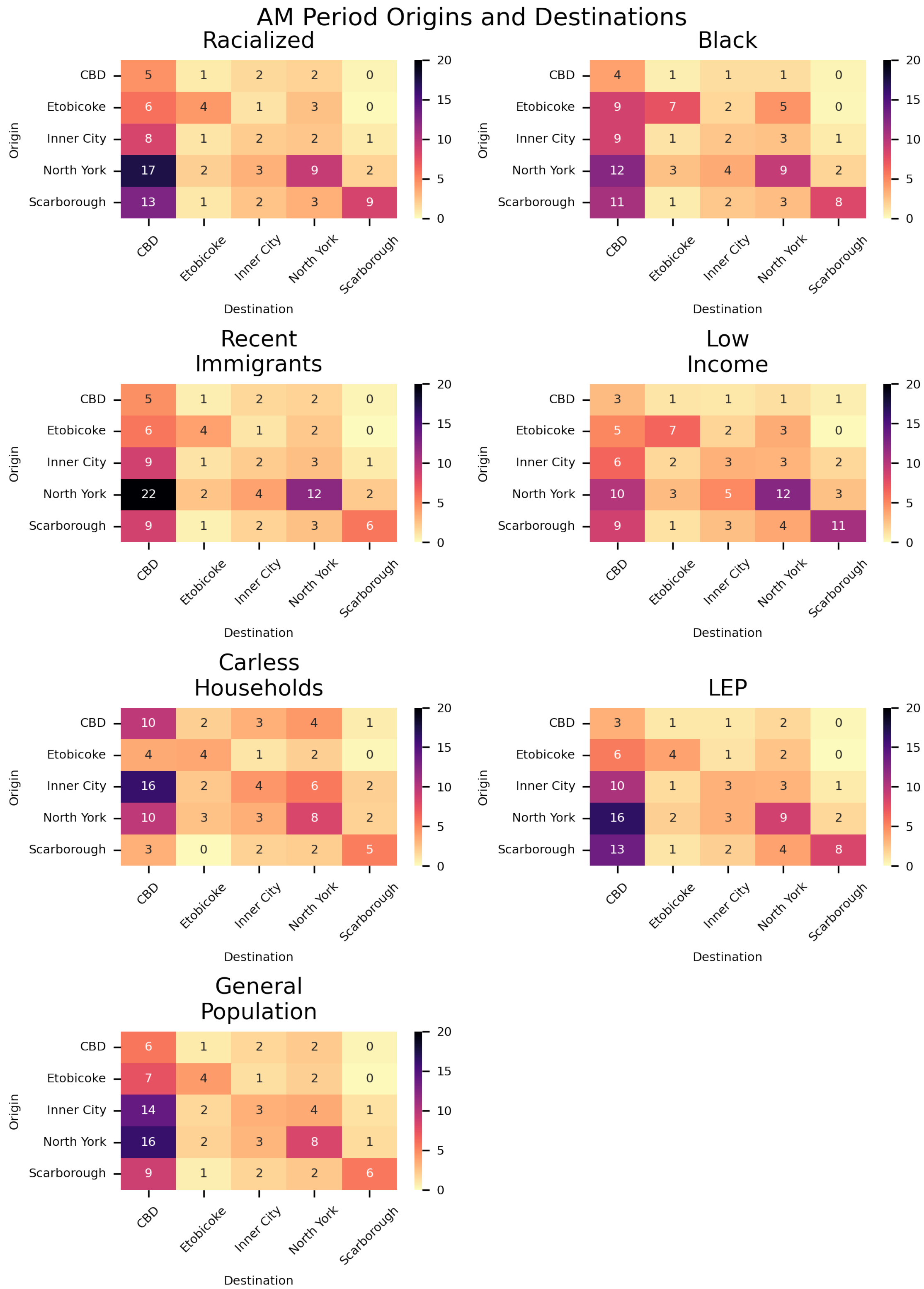

Figure A.6: Trip flow heatmaps by district for the AM period.

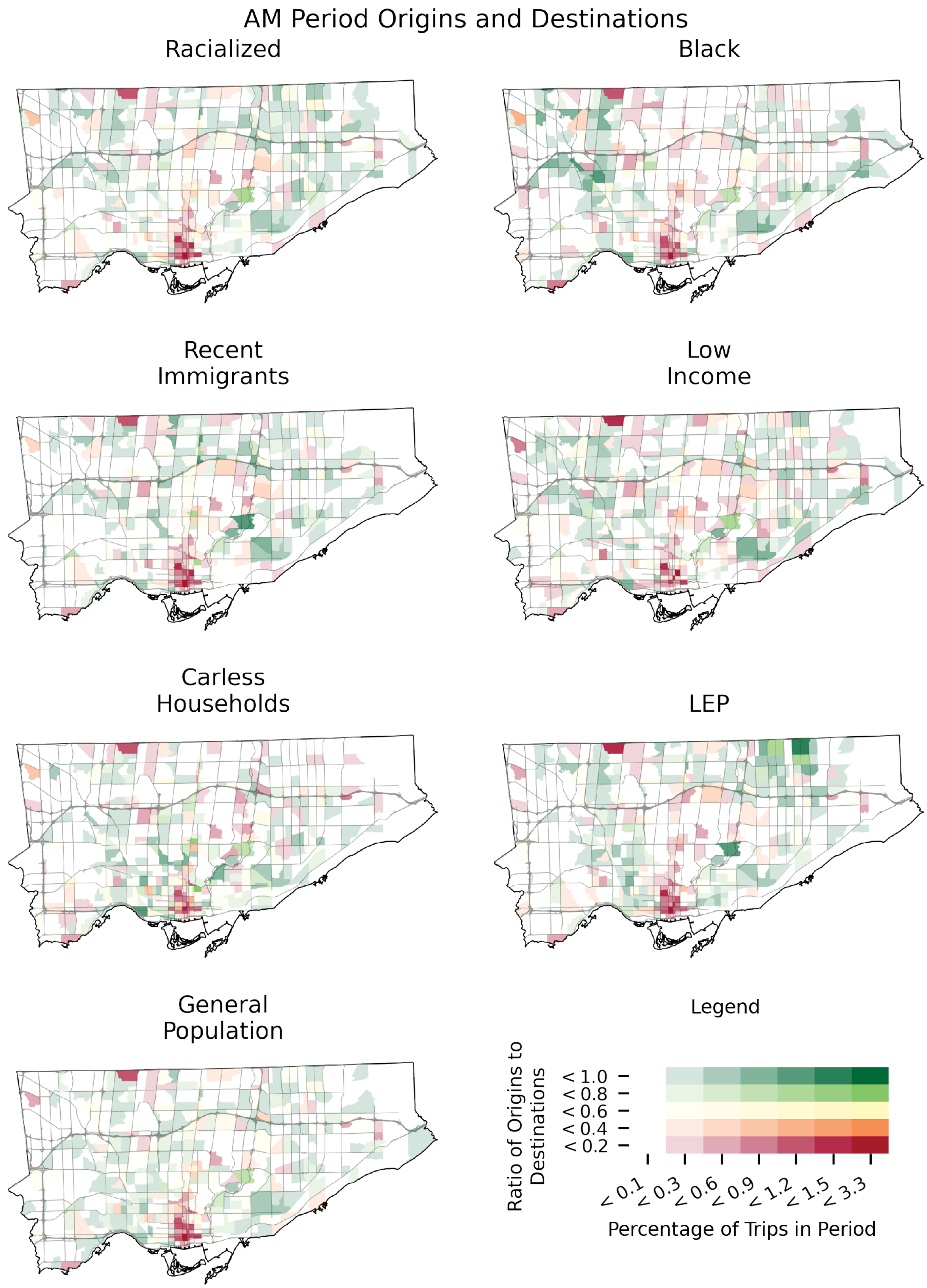

Figure A.7: Bivariate OD trip flows by TAZ for the AM period.

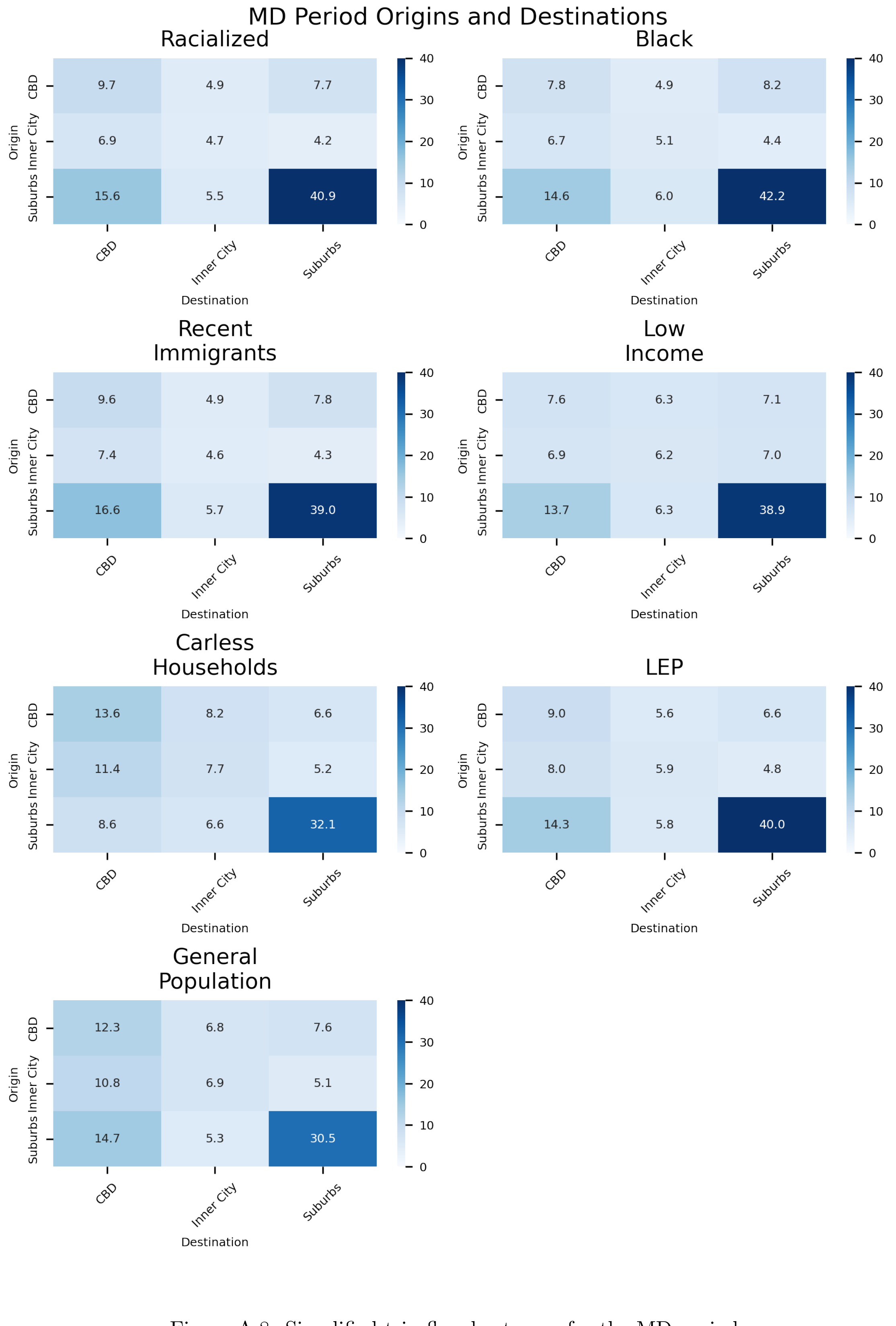

Figure A.8: Simplified trip flow heatmaps for the MD period.

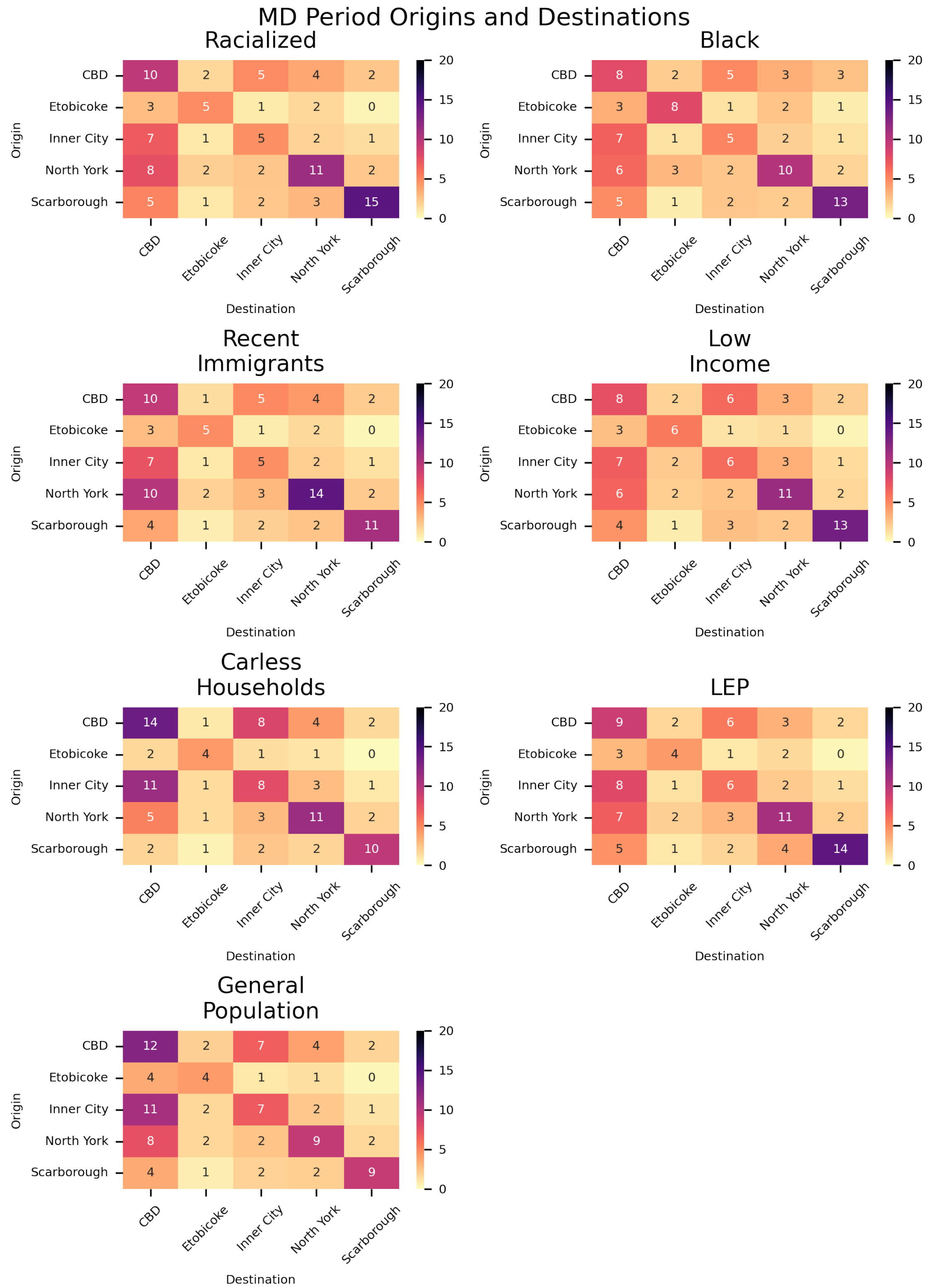

Figure A.9: Trip flow heatmaps by district for the MD period.

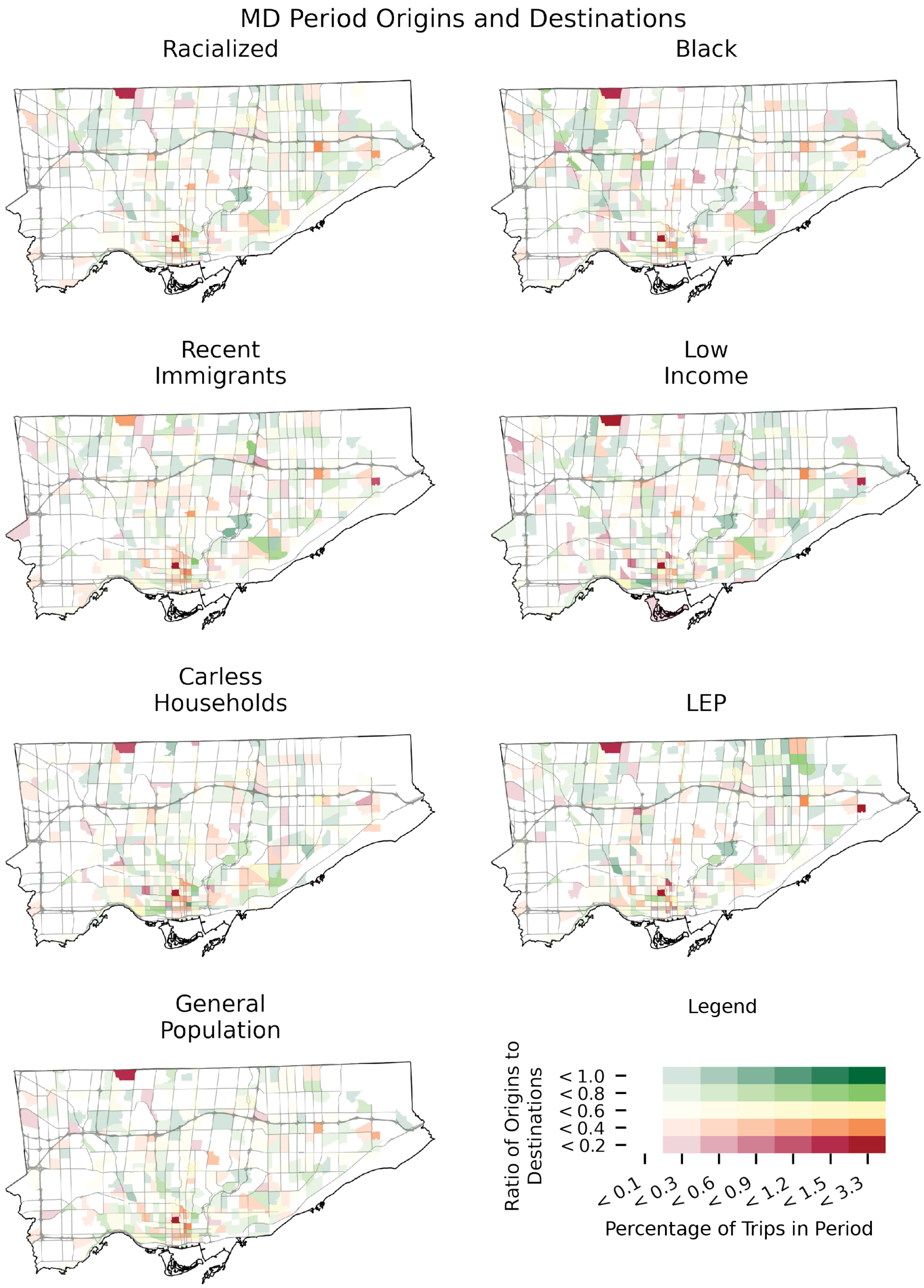

Figure A.10: Bivariate OD trip flows by TAZ for the MD period.

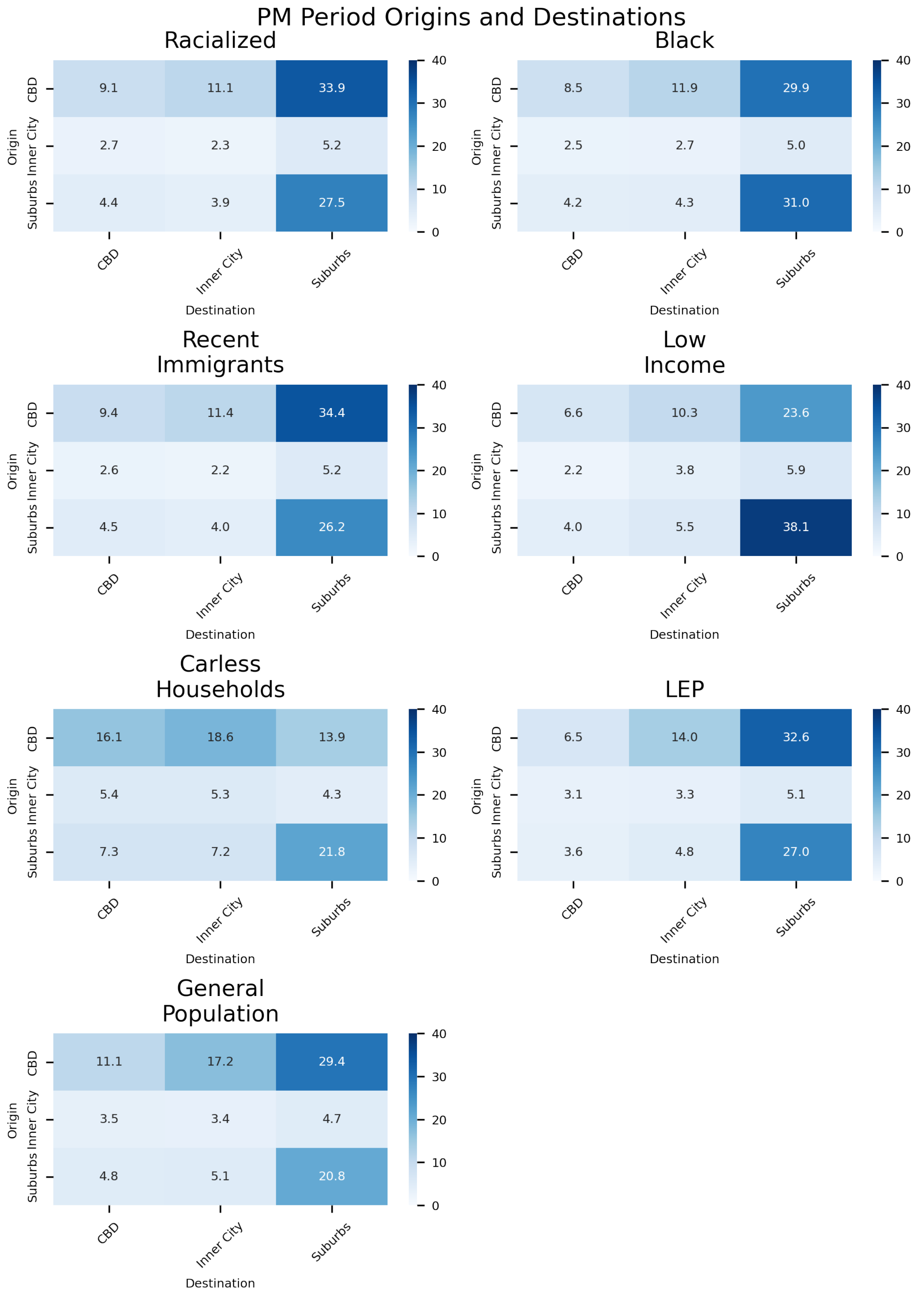

Figure A.11: Simplified trip flow heatmaps for the PM period.

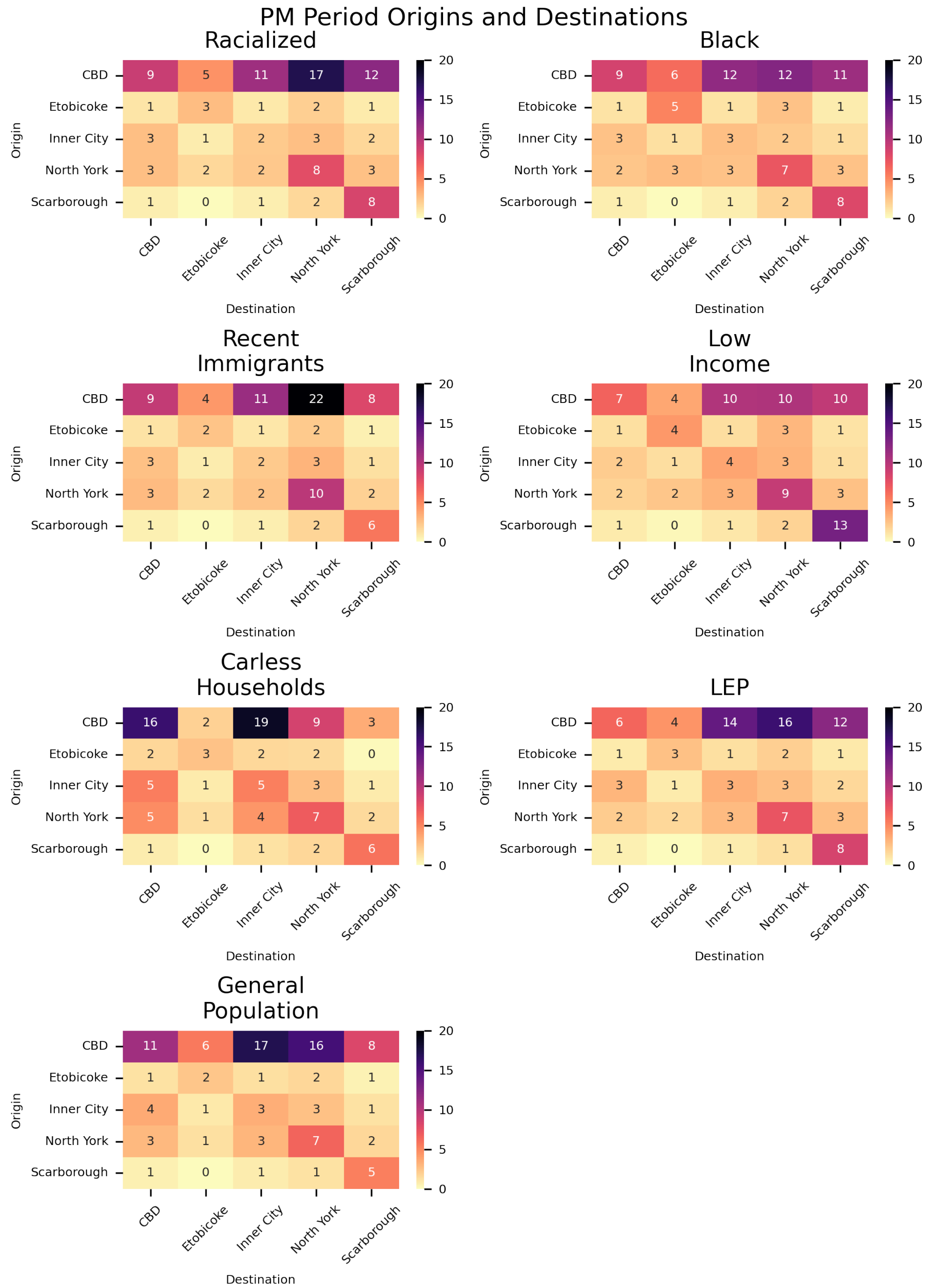

Figure A.12: Trip flow heatmaps by district for the PM period.

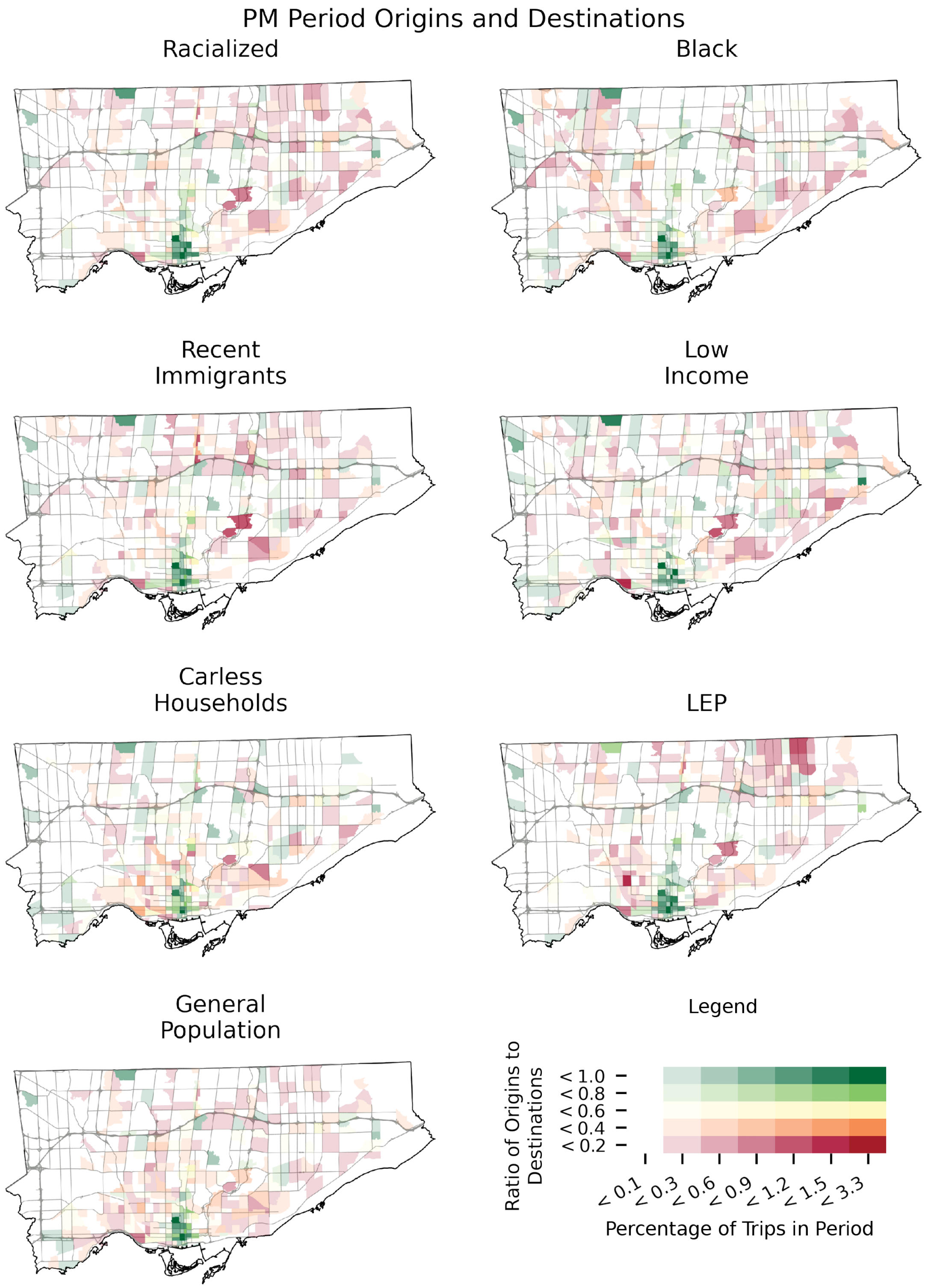

Figure A.13: Bivariate OD trip flows by TAZ for the PM period.

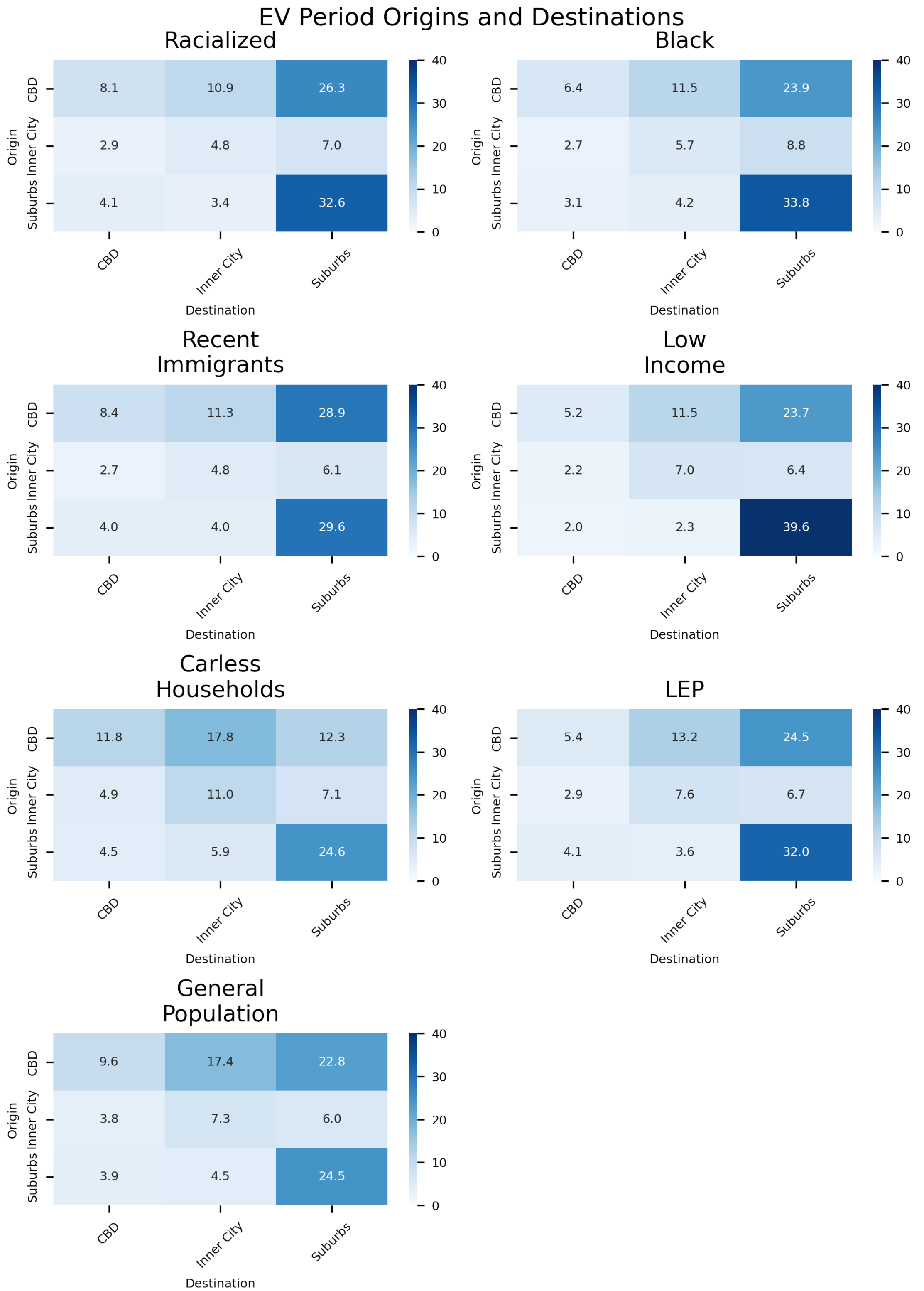

Figure A.14: Simplified trip flow heatmaps for the EV period.

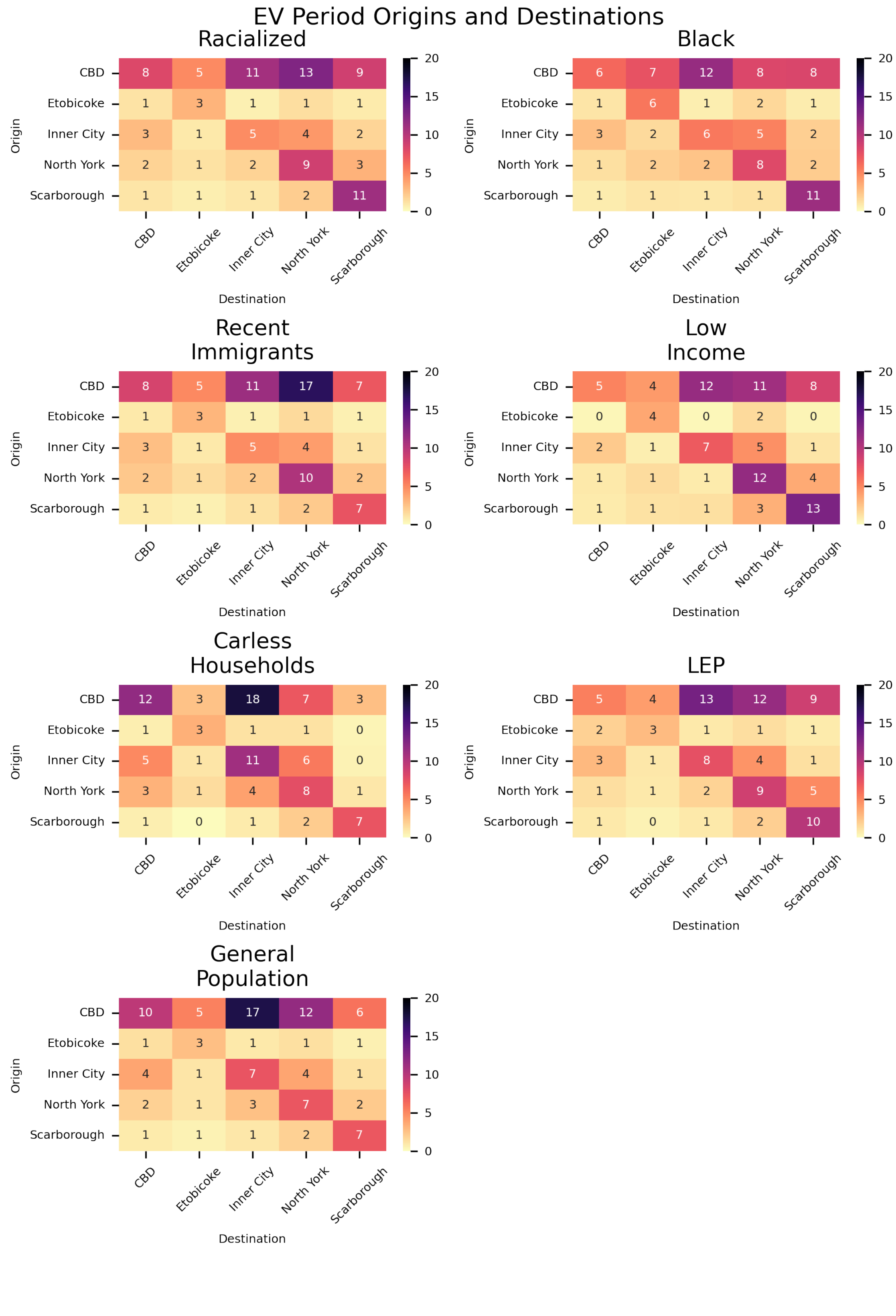

Figure A.15: Trip flow heatmaps by district for the EV period.

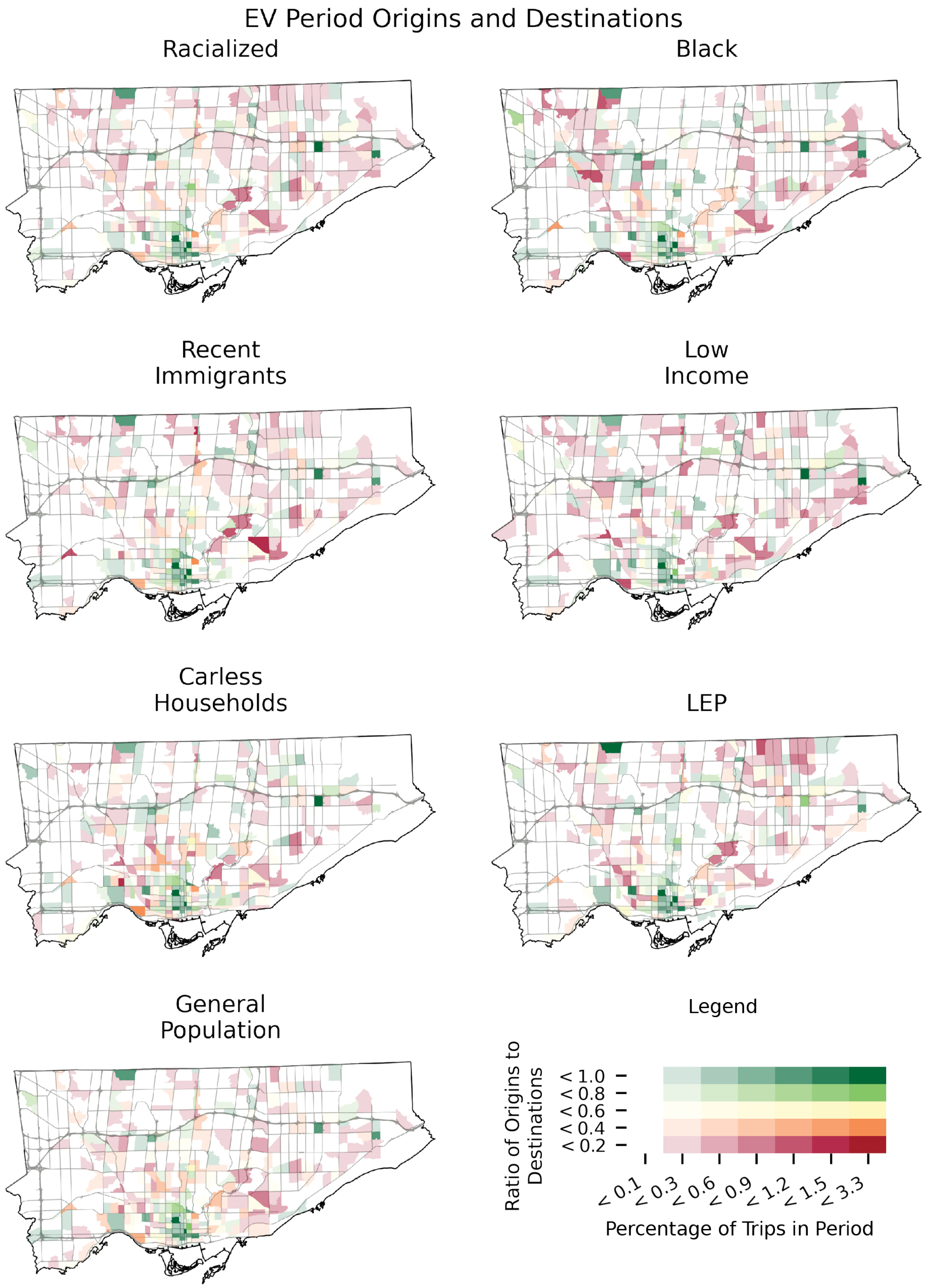

Figure A.16: Bivariate OD trip flows by TAZ for the EV period.

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