An Equity Lens on Public Transit System Vulnerability and Importance Using a Time-Expanded Network Approach

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ABSTRACT
We present a methodology to use a time-expanded graph for the evaluation of betweenness centrality, and the general concept of importance. We will also apply social equity concepts to the importance analysis. The time-expanded graph allows for an accurate consideration of transfers, and low frequency transit routes. In a case study involving the City of Toronto, we developed a population weighted betweenness centrality measure where the weight is the number of riders travelling from an origin to a destination, belonging to a certain group, enabling us to examine transit equity implications. Through our analysis, we find that equity-seeking groups, such as racialized users, recent immigrants, and low-income users, are more concentrated on fewer nodes and edges than the general population, and as such they will be disproportionately affected if those hubs are disrupted. We also find equity-seeking riders are more likely to be concentrated on frequent bus routes, rather than Toronto's streetcar or subway network.

INTRODUCTION

In recent years, the transportation literature has dedicated more attention towards the topic of transportation equity as society has become more cognizant of the struggles that the most vulnerable users experience in their daily lives. In contrast to the notion of equality, equity suggests that the most vulnerable members of society should be given better transportation opportunities than the general population. The COVID-19 pandemic has amplified the need of addressing transit equity issues, as those who are racialized, have lower incomes, and lack access to a vehicle, rely heavily on transit through the pandemic and will be the first ones to return to transit during the recovery (1).

Previous equity analysis in the public transit context has focused traditionally on the concept of accessibility, which represents the number of opportunities reachable from an origin within a time, and possibly cost, budget. A few efforts have recently focused on the interplay between equity and network structure (2–5), and bus reliability (6,7), offering useful insights and showing much promise. This study is an extension to this line of research.

Service reliability remains a key consideration in the attractiveness of transit (8). It refers to the ability of transit service to remain consistent in the face of recurrent perturbations present in the operating environment (for example due to interactions with the general traffic or surge in transit demand), and it is often measured in terms of on-time performance of individual routes. In contrast to recurrent perturbations, service disruptions in emergency scenarios occur less frequently but have far more severe and prolonged impact on service, resulting in the temporary loss a user’s primary path, with no competitive paths available for use alternatively. This concept is often described as vulnerability or generally resiliency (9).

Previous research on transit vulnerability and resiliency has primarily focused on metro networks (9,10), either by excluding surface transit in their graph models, or assuming surface transit can be represented much in the same way as metro networks. This ignores the role and uniqueness surface transit has in transit networks. In most transit agencies, the bus network contains a greater share of ridership than the metro network (11), and many of those routes do not operate at high frequencies throughout the day.

We present a time-expanded graph approach to represent a multimodal transit network for graph analysis. This method presents a realistic representation of transfers while reducing graph size. It also sufficiently captures the role headways have on the network. We also adapt betweenness centrality to consider origin-destination flows and apply this measure across different equity-seeking groups. Through this method, we conduct an equity analysis on the importance of nodes and edges, and whether equity-seeking groups are more concentrated spatially in their travel patterns than the general population.

LITERATURE REVIEW

Transit Equity

For society’s most vulnerable users, mobility provides a key avenue to improve their daily lives. Public transit has an important role in providing mobility since it increases the number employment opportunities they can access over simply walking (12). Transit also has a role in reducing social exclusion since it allows equity-seeking riders to access essential activities such as groceries, education, health clinics, recreation, and entertainment (13).

The idea of equity, which may be labeled more precisely as vertical equity, is different from equality, which might also be labeled as horizontal equity. Equality is premised on the basis that all parties should receive the same level of opportunities, while equity is rooted in the basis that to reach the same outcome, different levels of opportunity have to be provided (12,14). In the public transit context, this might manifest as providing more service hours and more reliable services to neighbourhoods populated by equity-seeking riders, such as low-income or racialized residents. Some studies, particularly those involving a Lorenz curve or Gini coefficient to analyze the equality of service levels (4,14,15), conflate equity with equality, although equal service is not necessarily equitable.

Besides Lorenz curves, there have been many transit equity studies that analyze accessibility, typically measuring access to opportunity (jobs, essential services) within a specific travel time window. Many studies aggregate neighbourhood-level results into quantiles defined by social equity indicator quantiles (13,16–19), such as household income, proportion of recent immigrants, or a composite index of many indicators. Other accessibility studies present results disaggregated each equity-seeking group of interest such as comparing results black transit riders versus white riders (20).

Accessibility is not the sole consideration for an equitable transit system. Reliability is also important since equity-seeking users can face higher penalties for being late to their jobs than those who have more stable employment, and adapting to unreliability might decrease access to opportunities (21). Some studies have attempted to address the role reliability has in creating an equitable network; one study analyzed on time performance among routes served by disadvantaged and advantaged users (6). Another analyzed the equitability of the response to subway delays and subway disruptions (7). However social equity studies on reliability remain rare.

Vulnerability in Public Transit

Reliability is very important to the overall transit experience of riders, as unpredictable travel times can lead to riders to add an unnecessary buffer (22) to their trips, which would lower their overall access to potential opportunity. Reliability typically captures variation in transit level of service at the trip level, and involves perturbations and variations of travel time due to common events; disruption is to reliability but different in that the events causing disruption are less frequent and can lead to shutdowns in service ranging from multiple hours to multiple weeks, depending on the severity of disruption (23). Vulnerability and importance are concepts that measure a network's susceptibility to disruption. A vulnerable network tends to have disruptions in service that will last for longer durations than an unreliable network (23), and will cause significant reductions in network serviceability (24). Similar to vulnerability is the concept of importance. Importance is defined as how important a transit route or stop is to the entire network, and the level of disruption it will cause if negatively affected (25,26).

Graph Theory Approaches to Vulnerability Analysis

Graph theory offers the most common approach to the analysis of network vulnerability or importance. The birth of Graph theory is attributed to Euler while addressing "The Seven Bridges of Konigsberg" problem in the 18th century. Graph theoretic methods represent intersections, stations, or bus stops as nodes, and sections of roads, rail lines, streetcar tracks, or bus routes as edges (10). Some graph analyses make the distinction between a P-space, where nodes directly connect with all other nodes and the edge cost is the travel time between individual nodes, and L-space, where nodes are only connected if they are adjacent to another on a transit route in the transit network (27,28). There is also the distinction of planar, where edges must intersect at nodes, and non-planar, where the graph is three-dimensional and edges can cross over other edges without intersecting (10).

A common measure to analyzing vulnerability and importance of a node or edge is betweenness centrality (5,29–31), which can be defined by the following equation(32).

$$g(i) = \Sigma_{o \neq d} \frac{n_{od}(i)}{n_{od}} \quad (1)$$

where

n_{od} = number of shortest paths between an origin o, and destination d

$n_{od}(i)$ = number of shortest paths between an origin o, and destination d, crossing node i

$g(i)$ = betweenness centrality

Betweenness centrality can also be normalized by the maximum betweenness centrality for an equivalent network with the same amount of nodes (33). This can be expressed as the following for a directed graph.

$$normalized\ g(i) = \frac{g(i)}{(N-1)(N-2)} \quad (2)$$

where

N = number of nodes in the graph

A higher betweenness centrality for a specific node or edge would indicate that such node or edge has greater importance to the network (32), and its removal would make the network more vulnerable. Other types of centrality measures exist, such as eigenvector centrality, degree centrality and closeness centrality, which uses other variables instead of travel time and shortest paths (34).

Graph Theory Approaches in Public Transit

One weakness of the traditional graph approach to analyzing a public transit network is the assumption that the nodes and edges, representing stops and transit route sections, exist at the same service level at all times. While this may be a reasonable assumption to make for high frequency services such as metro networks, it overestimates service in outlying areas which are typically covered by low-frequency surface transit. In contrast to classifying graphs by P-space or L-space, Whited developed an alternative method to classifying graph networks in the transit context using the labels of route-map, trip-map, and time-expanded (31).

Route-map graphs represent each route as a set of edges and stops as nodes, similar to an L-space representation, while a trip-map graph plots each trip as a set of parallel edges, usually in a multi graph (31). A time-expanded graph transforms a temporal graph into a static graph by producing a copy of each node at each instance of time; edges are then drawn between each time-node combination at time instances whenever the edges exist (35). In the public transit context, this would mean that edges connecting stops would only exist when the timetable indicates a bus or train is travelling between the stops. This representation allows a static graph to take into consideration the aspect of headways and frequency of service.

Various authors have attempted to integrate headways into graph analysis. For an analysis of the public transit network of Vancouver, Qunitero-Cano incorporated headways into a route-map graph by assigning a frequency factor into each edge of the graph (36). The factor would then be used as a weight in the computation of common graph measures such as complexity, and connectivity. Maduako used a different approach in creating up to 165 daily snapshots of the transit network in Moncton, to account for the changes in the transit network for different departure times (37). Both Whited and Fortin implemented the time-expanded graph for an analysis into the networks of Edmonton, and Chambly, Quebec, respectively (31,38). However, the use of time-expanded graphs in public transit contexts remains sparse. Transit networks that have many low-frequency routes, such as buses or commuter rail, stand to benefit the most from using a time-expanded approach. Previous applications of graph theory focus mainly on metro, light rail and bus rapid transit networks (9,10), which may be why time-expanded graphs were not common or needed. However, for graph analysis specifically involving bus networks, some studies represent bus networks as a L-space network similar to a route-map graph (34,39,40), and avoid considering the impacts of headways.

Project Novelty

With previous applications of graph theory in for vulnerability focusing on representing the network as a static graph, we see an opportunity to address these. The time-expanded model will consider the headways of individual transit routes, and we can accurately model transfers so that low frequency transit routes are less desirable to transfer onto, much like the path finding behaviour of passengers.

We also see a lack of integration of social equity into the concepts of importance. These groups face the greatest barriers to economic advancement. We will use the time-expanded graph to apply a betweenness centrality analysis to determine whether equity-seeking riders are more vulnerable to disruptions than the general population.

This is important because many equity-seeking populations are reliant on local buses, and low frequency routes as a last mile solution to reach rapid transit station, and tend to live further away from the subway network (41). By not considering headways, previous studies may assume those bus routes are more resilient and less vulnerable than reality.

METHODOLOGY

The study requires census data, GTFS feeds, an OD travel survey, and road network GIS files as inputs. Census data and the travel survey were used to synthesize populations, while the GTFS feed and GIS data were used to build the time-expanded graph. These processes are shown in Figure 1. We chose to apply this analysis in a case study involving the City of Toronto.

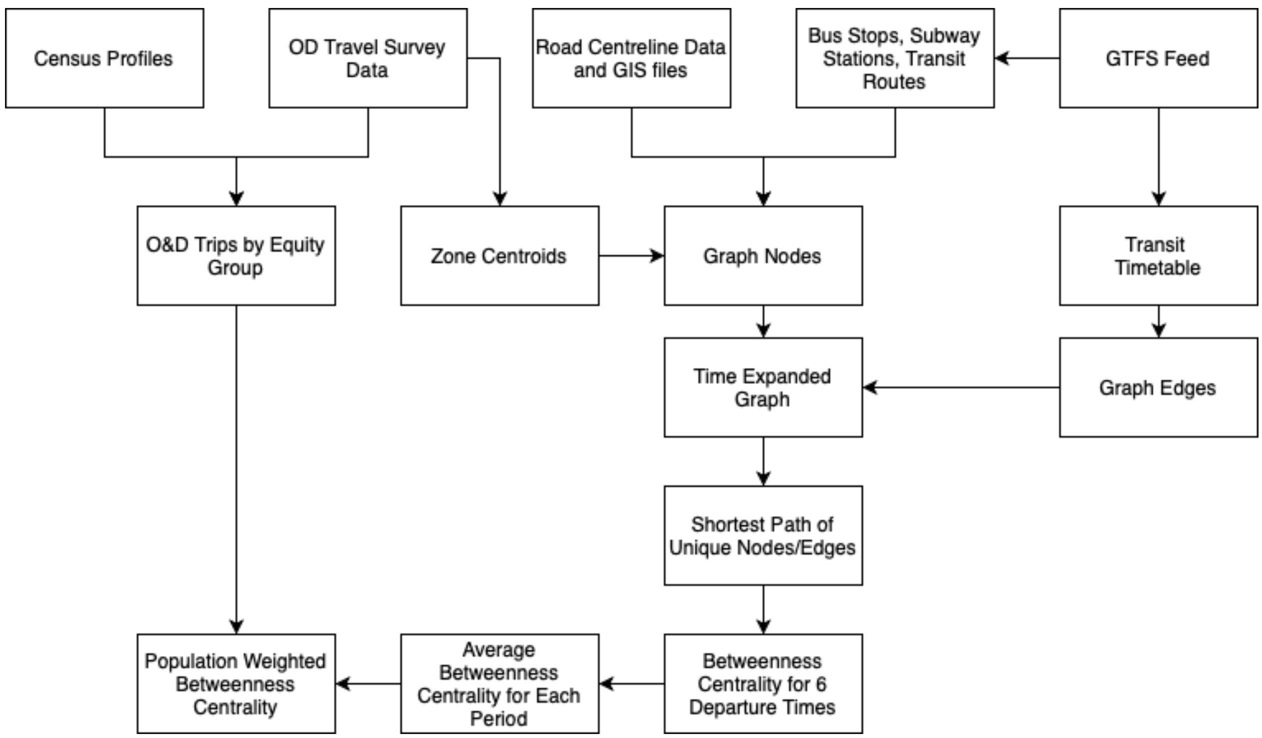

Figure 1: Simplified flowchart of the methodology for the analysis

Population Weighted Betweenness Centrality

As described previously, betweenness centrality is a measure of importance that measures the proportion of shortest paths crossing a node or edge. However, a criticism is that the measure fails to take into account the usage of a stop or route (25); for example while two stops might have the same betweenness centrality value, the stop with higher ridership is more important and will increase vulnerable if disrupted. Therefore, we introduce a population weighted betweenness centrality (PWBC) measure, shown in equation 3, where the measure is weighted by the number of trips from a specific origin to a specific destination, over the total number of trips.

$$g_{pop}(i) = \frac{\sum_{o\neq d} \frac{x_{od} n_{od}(i)}{\sum_{od} x_{od} n_{od}}}{(N-2)(N-1)} \tag{3}$$

where

x_{od} = number of trips using transit from origin o to destination d

n_{od} = number of shortest paths between an origin o, and destination d

$n_{od}(i)$ = number of shortest paths between an origin o and destination d, crossing node i
N = number of unique intersections or subway stations in the graph
$g_{pop}(i)$ = population weighted betweenness centrality

The population weight can alternatively be defined by equity-seeking group, such as the total number of trips made by recent immigrants, instead of the total number of trips. This method allows us to find nodes or edges specifically important to each equity-seeking group and explore how this differs from the general population.

While many graph packages have algorithms to calculate betweenness centrality, we did not find a suitable package that can calculate betweenness centrality in a time expanded graph. Instead, we calculated the betweenness centrality from computing all possible Dijkstra shortest paths between an origin and destination. Later, we will discuss how the time-expanded graph will have multiple nodes located at the same intersection, but to take this fact into account, we simplified the shortest paths to be a list of all unique intersections/subway stations and route segments visited, instead of a list of all nodes or edges visited.

Defining Equity-seeking Groups

Our method requires an OD travel survey. The Greater Golden Horseshoe region, encompassing the City of Toronto, conducts the Transportation Tomorrow Survey (TTS) at a 5% sample size, with the most recent iteration being conducted in 2016 (42). One resident per household is asked about the details of all trips made by members of the household on the previous weekday. Respondents also provide other demographic data, such as the household size, age and gender of each household member, household income and the number of residents in the household. However, other demographic data, such as ethnicity or immigration status, was not collected in the survey. Results were aggregated to the nearest TAZ and the most recent survey took place in 2016.

Based on the guidance of the TCRP Research Report 214 (43) and previous literature, we chose to use the following equity-seeking groups for the analysis: racialized residents (composed of visible minorities and indigenous residents), recent immigrants landing between 2011 and 2016), residents who have no proficiency any official languages, low-income residents, and carless households. We also specifically analyzed black residents, which would be a subset of racialized residents. The number of residents in carless households and number of low-income residents making trips can be pulled directly from the TTS, while Census data was used to determine the populations of the other groups.

To determine the number, origin, and destination of trips made by other equity-seeking groups, publicly available census profiles was used. These profiles only include the number of men and women for each census question, such as the number of recent immigrant men and number of recent immigrant women.

TAZs generally have populations ranging between 5,000 to 15,000, while the smaller census dissemination area (CDA) has populations ranging between 0 to 2,000. Since TAZs do not perfectly match up with CDAs, we assumed that population density and composition was uniform across the entire CDA. We then calculated the population of each equity-seeking group in each TAZ by taking a weighted average of the population for each CDA inside the TAZ. This population can then be used to create a proportion of the total population that falls into each group we defined for the analysis. We then used the number of males, females, and their household locations to match the TTS and census data to estimate the number of transit trips made by each equity group from an origin zone to a destination zone. The resulting numbers of trips (Figure 2) were then used to weight the betweenness centrality to create the PWBC measure.

This approach ensures that the analysis takes into consideration equity-seeking riders who use transit from origins and/or destinations outside their household location, such as non-home based trips (44). While an effort was made to estimate the number of indigenous riders in the analysis, their low number disallowed their consideration as a separate group in the analysis.

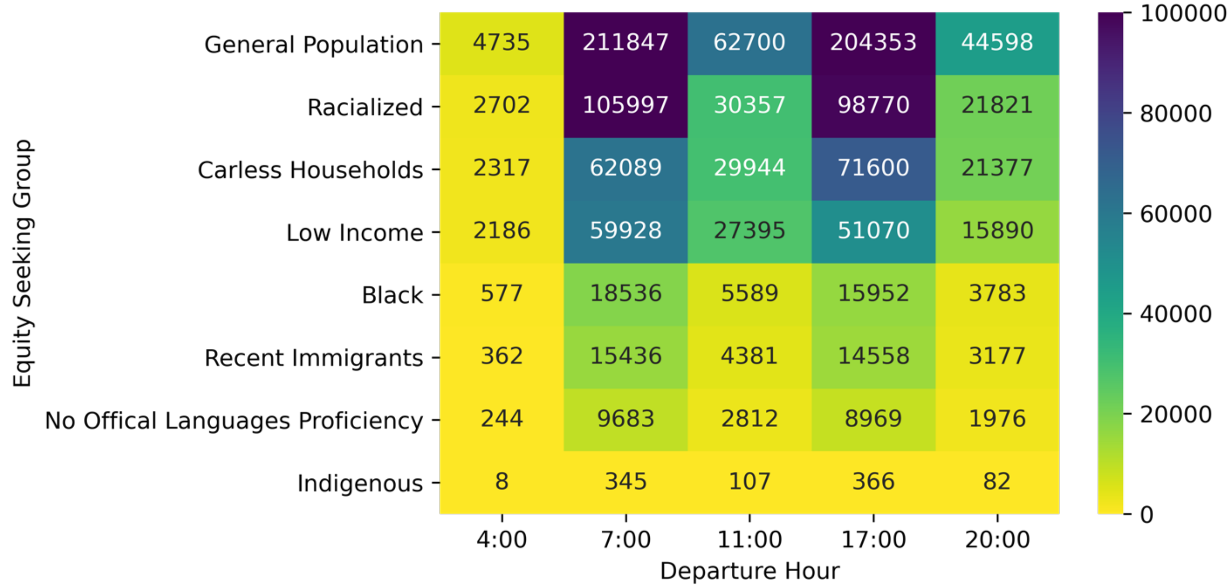

Figure 2: Estimated number of transit trips made by each equity-seeking group for each departure hour

Matching Stops to Intersections

GTFS files were required for the location of stops, and transit timetable. For our case study, the October 2016 GTFS feed for the Toronto Transit Commission was used as the basis to build the time-expanded graph. To reduce the size of the graph and simplify the paths that involve a transfer from one route to another, stops were matched to either intersections or subway stations. The intersections or stations were used as the graph nodes instead of individual bus stops. This process eliminates the need to build walking edges between stops for transfers. Only city defined "Major Intersections" and "Minor Intersections" were used for matching. As a result, we did not include mid-block bus stops in the graph, which accounted for a very small proportion of the roughly 10,000 stops in Toronto.

To match a stop to an intersection, the stop must be roughly within 75m to 200m of an intersection or subway stop, depending on the type of intersection, subway station, or streetcar loop. Many nearside and farside stops were within this range, but some manual adjustment was done on some intersections to ensure that all stops were appropriately assigned to an intersection or subway station (Figure 3). Only intersections that had at least one stop assigned to it, along with all subway stations, were used to build the graph.

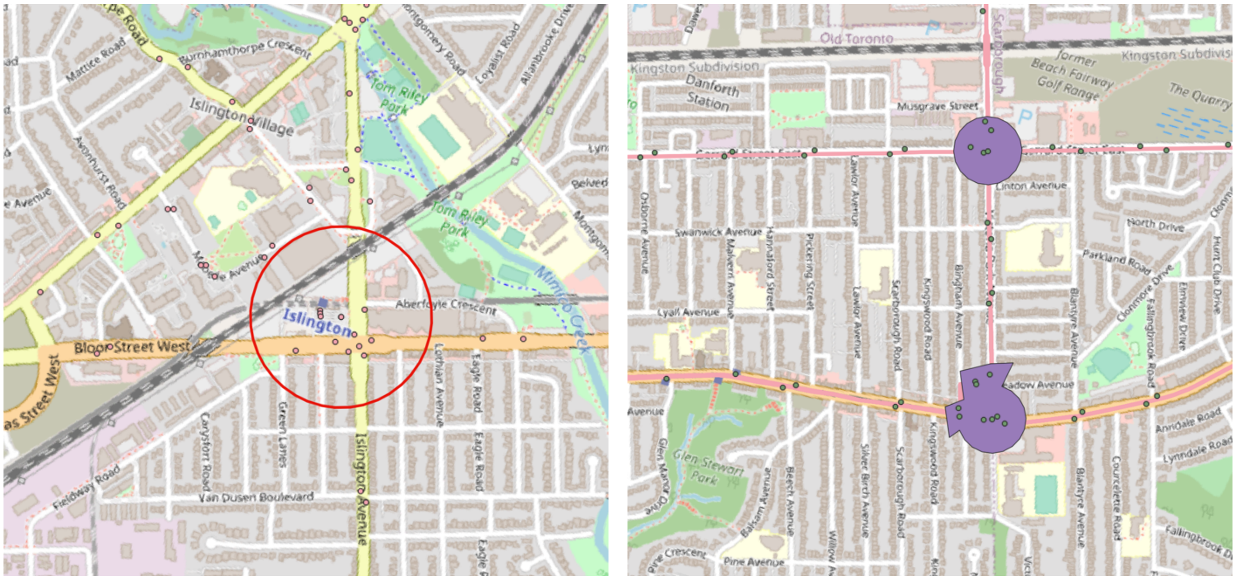

Figure 3: Examples of matching stops to stations (left), intersections (right), and manual adjustments for the matching (right). Bus stops are represented as dots.

Graph Build Procedure

For the time-expanded graph, all edges were directed. Intersections and stations were added as nodes to the graph, and edges, representing route segments from the GTFS, were then drawn between the intersections. In cases where multiple stops of the same route are assigned to the same intersection or station, the stop located the closest to intersection or station was used as the representative stop for edge travel times. Other nodes that were added were TAZ centroids, representing the set of origins and destinations.

We made a distinction between index nodes, and route specific nodes, both of which are located at the same intersection or station and physical space. Edges that represent transit route segments connect route specific nodes to other route specific nodes of the same route, and the cost was based on the travel times found in the GTFS. Each route specific node connects to the index node for that intersection via boarding and alighting edges, at a cost of 2 minutes. This simulates the transfer time transit users need to walk across an intersection, or within a subway station and sets a minimum transfer time of 4 minutes. This also reduces computation overhead, since it adds a cost to waiting, and reduces the number of possible shortest paths, especially for low frequency routes. 4 minutes was also the time used by other transit routing services, such as Google Maps (45).

That index node of a given intersection also connects to TAZ centroids within 30 minutes walking distance, and the cost for that edge was the walk time. To simulate waiting and transfers, the index connects to another index node at the same intersection one minute in the future.

One minute was chosen as temporal resolution of the graph as smaller time increments would increase computation and memory requirements. All travel times were rounded to the nearest minute for the route specific edges.

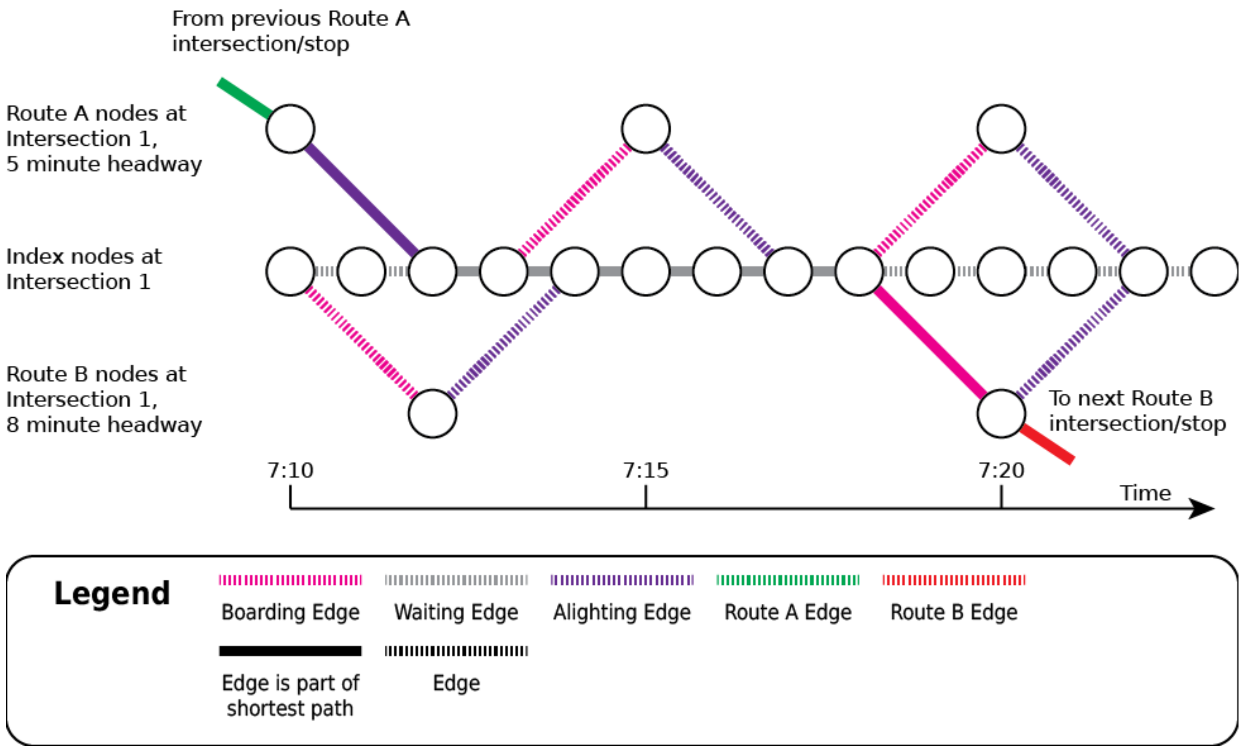

Figure 4: Example of a shortest path involving a transfer at intersection 1.

In the simplified example seen in Figure 4, there is a ten minute transfer at intersection 1 between route A, which has a five minute headway, and route B, which has a ten minute headway. The transfer is composed of one alighting edge representing two minutes, one boarding edge representing two minutes, and six waiting links representing six minutes. While route B services the intersection at 7:12, this would not be a feasible transfer because there would not be enough time to alight, and then board the connecting bus. This is represented in the directed graph by the fact there would be no series of edges connecting the route A arrival at 7:10, and the route B departure at 7:12; instead, the 7:20 departure is chosen for the shortest path.

Betweenness Centrality Computation

We analyzed results for six periods (Figure 5), with six departure times inside each period. These departure times were all inside the first hour of each defined period, defined as the departure hour. On routes with very long headways, the shortest path and shortest path length can vary significantly depending on the departure time chosen, which can significantly affect the analysis (46). Shortest paths were calculated at six random times in 10-minute blocks within the departure hour instead of a single time much like the process used by Stępniak (46); as an example, for the AM period, shortest paths and betweenness centrality were calculated at 7:02, 7:18, 7:25, 7:30, 7:49, 7:56. This randomization was done for all other periods. The graph was built for five hours after the start of the departure hour, to ensure that as many origin-destination pair returns a shortest path. This meant for an intersection serviced by 1 route at a 10 minute headway in each direction would roughly have 360 nodes representing it, representing 300 minutes' worth of nodes for the index node, and 30 nodes representing each instance a bus arrives at the intersection, in each direction.

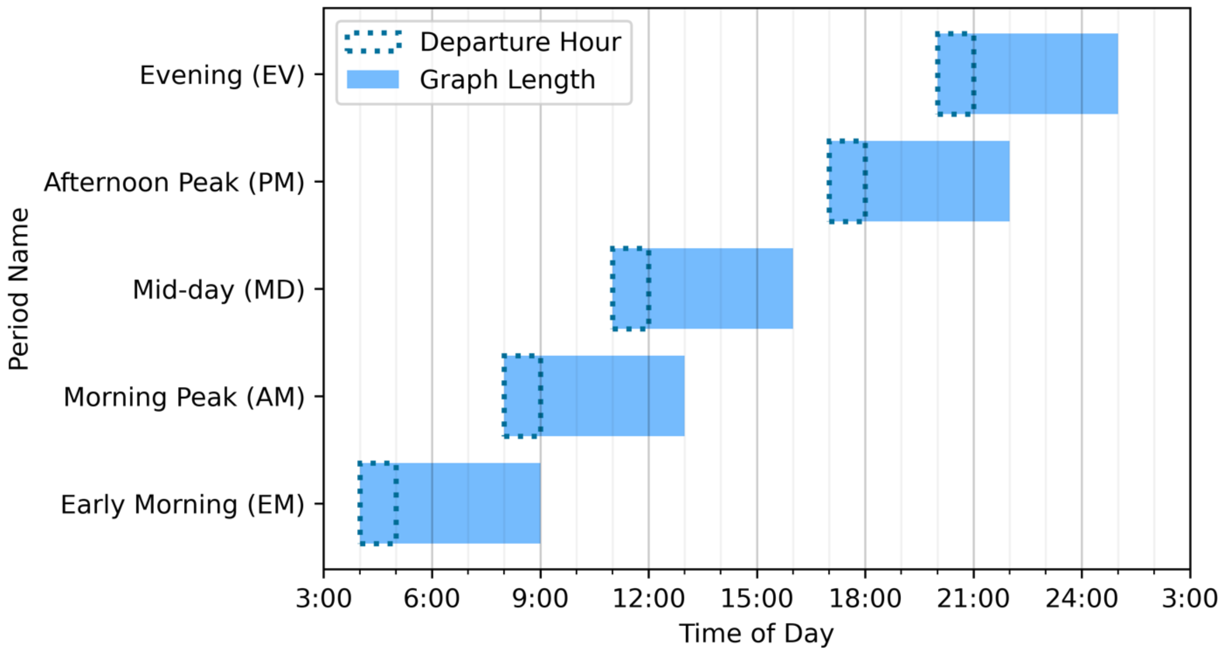

Figure 5: Departure hour and graph duration for each period

RESULTS

Toronto Case Study

As mentioned previously, we will apply our methodology to the City of Toronto. The Toronto Transit Commission (TTC) is Canada's largest transit system by daily ridership (11), and second busiest among systems in Canada and the United States, behind only the MTA in New York City. The TTC provides transit service on buses, streetcars, exclusive right-of-way streetcars, and subways. Toronto's high frequency trunk bus routes, termed the 10-minute network by the TTC due to the fact they run at all day 10-minute headways except overnight, roughly operate on a grid that connects to the city's subway lines, and many of the 10-minute network routes run overnight as part of the Blue Night network. The city's subway network does not run overnight, but it is replaced by buses in the Blue Night network. The city's bus and streetcar ridership constitute 64% of the daily ridership, with the subway making up the rest (11).

Variability in Betweenness Centrality

To first verify the hypothesis that the PWBC varies by departure time, we took the coefficient of variation of the six calculated PWBC values for each node, weighted by the total number of transit trips in Toronto for each period. This analysis was done for all six periods. The coefficient of variation is defined as the standard deviation of the PWBC over the mean PWBC for a node. This measure was selected as it was normalized to the mean and can thus be aggregated with the results for other nodes.

$$CV_i = \frac{\sigma}{\mu} * 100\% \quad (4)$$

where

σ = Standard deviation of the six betweenness centralities within each period
μ = Mean betweenness centrality among the six departure times within each period
CV_i = Coefficient of variation for node i

The coefficient of variations was then plotted as a histogram for each period (Figure 6). We also categorized each node into 1 of 5 categories; inner subway station for stations located in the pre-1997 borders of Toronto, outer subway stations for other stations, streetcar for stops that are served by a streetcar route, frequent bus for stops served by a 10-minute network bus route and not served by a streetcar route or subway line, and bus for stops not served by streetcar, subway, or frequent bus routes. We plotted the coefficient of variations as box plots disaggregated by both the time period, and the category of node (Figure 6).

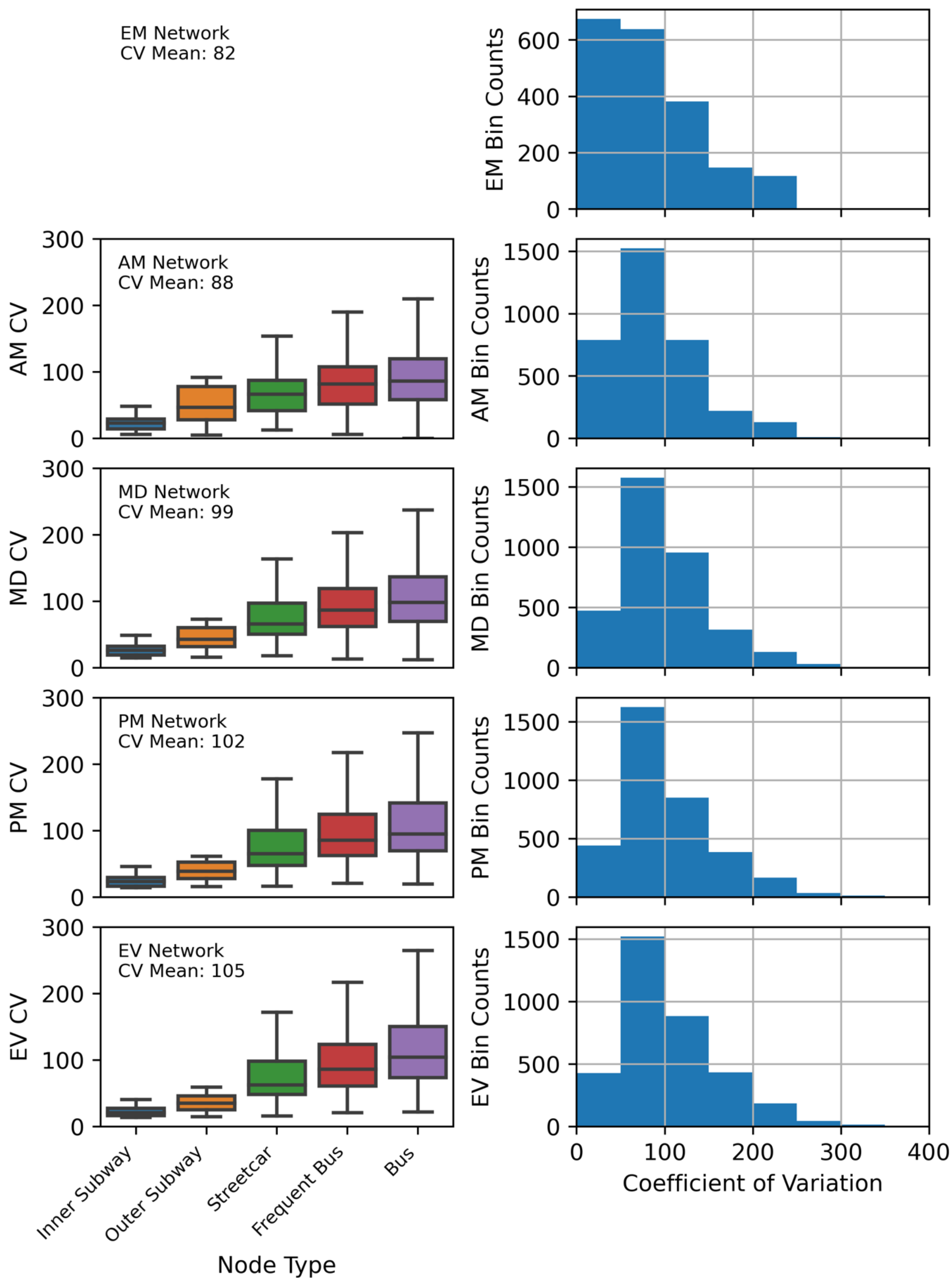

Figure 6: Boxplot and histogram for the coefficient of variation of betweenness centrality for each period. We did not disaggregate by transit mode for the early morning period since the subway does not operate during that departure hour.

From the results, many nodes had coefficient of variations over 100%, and all periods had a mean coefficient of variation above 80%. In a few nodes, the coefficient of variation had values close to 400%, showing very high variation in how important an intersection is to the transit network.

However, this result was not the same across modes. The coefficient of variation is relatively stable and low for both inner and outer subway stations. Outer stations have a slightly higher coefficient of variation because of lower frequencies on Lines 3 and 4 of the subway, and feeder bus routes connecting at the suburban stations. For the other three categories, the coefficient of variation is noticeably higher, indicating that the lower frequency increases the variability in the results. Non-frequent bus service had the highest variability since those routes had the longest headways, while downtown streetcar routes, which generally have higher frequencies, had lower variation.

What was surprising is the degree of variation, with the standard deviation consistently being above the mean PWBC. This analysis shows the risks at choosing a single departure. This is true even for frequent bus and frequent streetcar routes. Finally, this analysis shows the basis for choosing a time-expanded graph over an L-space graph or a route map graph, as the headway dynamics would not be captured in those graphs, as it would assume that shortest paths involving surface transit routes would always exist.

Entropy of PWBC by Group

Once we verified that averages of multiple departure times must be taken, we took the average of the six departure times for each period. We then weighted the average betweenness centrality by the number of trips made by each equity-seeking group. We calculated the entropy of the PWBC, where the entropy is defined as

$$E_{pop} = \Sigma_i \overline{g_{pop}(\iota)} * \ln\left(\overline{g_{pop}(\iota)}\right) \tag{5}$$

where

$\overline{g_{pop}(\iota)}$ = Mean of the PWBC for node i, among six departure times within each of the five periods

E_{pop} = Entropy of PWBC

The entropy would indicate the level of diversity in the PWBC; a higher value would mean a higher number of the specific equity-seeking transit users are concentrated on a few sets of bus stops or stations.

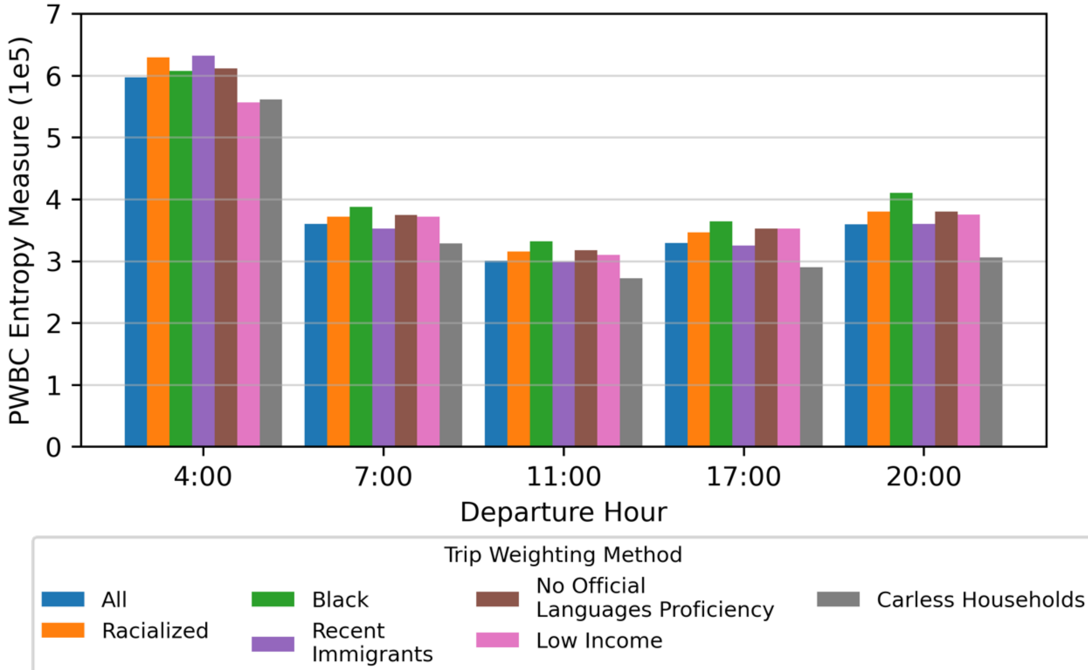

Figure 7: PWBC entropy measure by departure hour and group

For nearly all equity-seeking groups except carless households (Figure 7), the entropy was higher than the general population. This indicates that riders belonging to those equity-seeking groups were more concentrated onto specific routes, key bus stops, or subway station. Because those users are less likely to be equally distributed across the network, in the event a disruption occurs that disables those important nodes, they would be more vulnerable to disruption. The difference between the equity groups and the general population are greater in the evening periods compared to other periods. Of the equity-seeking groups, black transit users experience higher entropy, while carless households have consistently the lowest entropy. The entropy for the early morning period is higher than other time periods, which shows how the reduced overnight bus network concentrates potential trips onto a smaller set of routes and stops.

Mean of 95th Percentile PWBC Nodes by Group

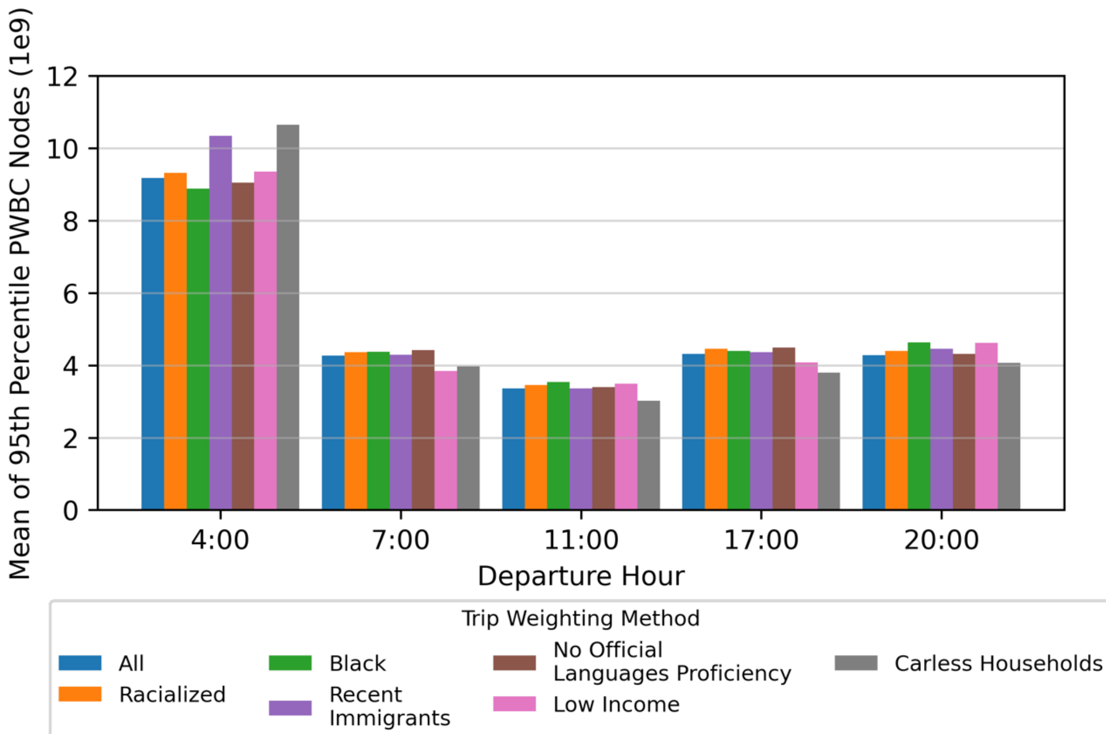

Figure 8: Mean PWBC measure by departure hour and group for nodes above the 95th percentile

For each equity group, we also computed the mean of PWBC values above the 95th percentile (Figure 8). The 95th percentile was independently calculated for each equity-seeking group. Since a high PWBC for a stop or station would indicate many trips and shortest path use this stop or station, this measure is intended to capture how critical the hubs are to each equity-seeking group. A 95th percentile was chosen to ensure that only important nodes were used for the mean, while ensuring that the chosen nodes were not entirely composed of subway stations. This typically resulted in a set of 50-200 nodes above the 95th percentile, depending on the time of day. Except for the early morning period, all but 3 subway stations along Lines 1 and 2 were above the 95th percentile for all equity-seeking groups.

The mean of the nodes above 95th percentile tells a similar story to the entropy measure. For nearly all equity-seeking groups and periods, the means are slightly higher than the general population. This reinforces the fact that equity-seeking riders are more likely to be concentrated onto a set of small stations or stops. The smaller difference for the means compared to the entropy measure may indicate that the PWBC distribution is heavily skewed and results in a few stops having large values.

Carless households remain the most distributed while black transit users are among the most concentrated for most time periods, however, this changes for the early morning period. This might indicate that carless household trips originate and end in a smaller geographical area; the number of nodes used by carless households in that period is small, but each node has a higher PWBC value, which explains why carless households have a high mean but low entropy for the early morning period.

PWBC by Group and Transit Mode

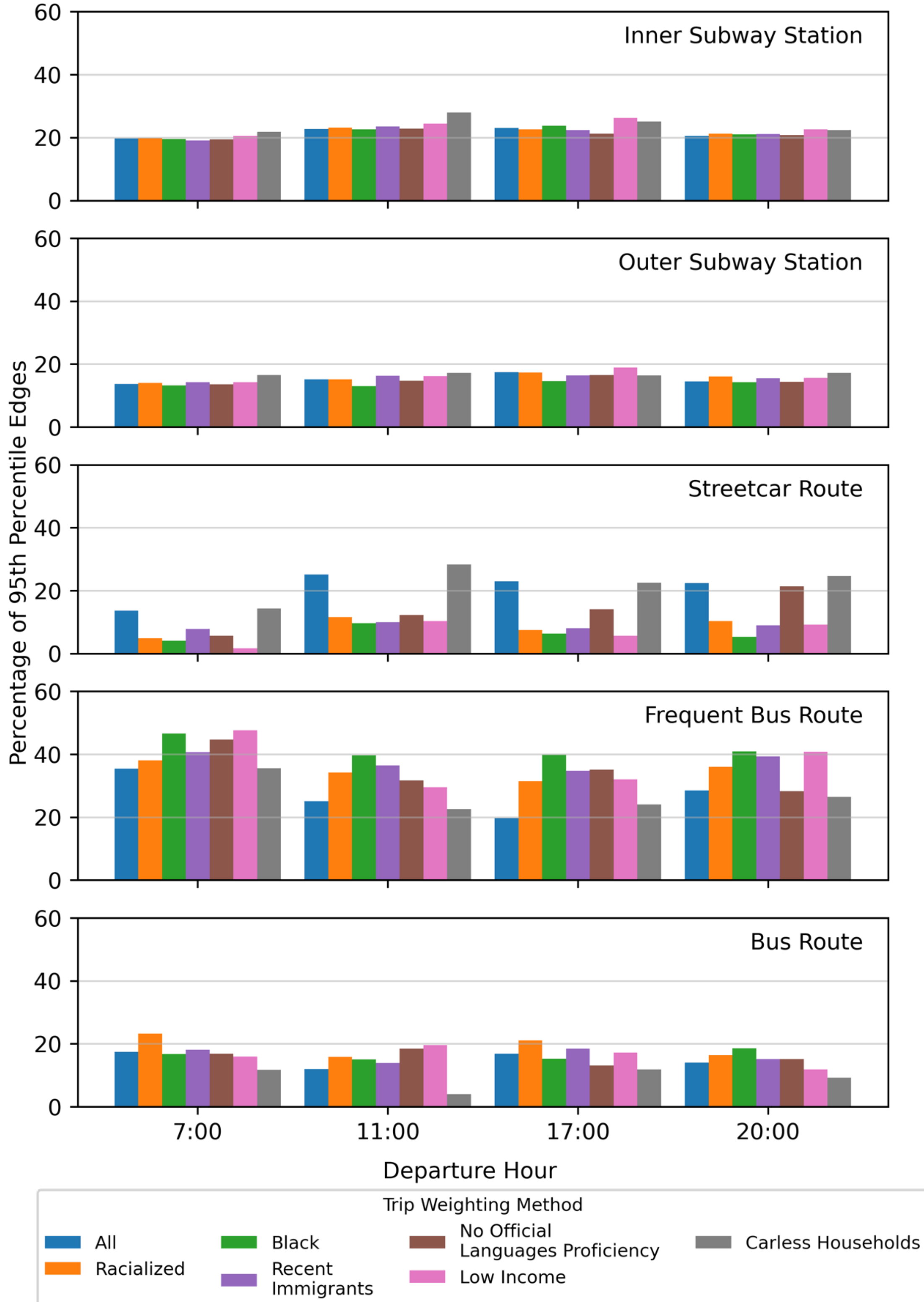

Figure 9: Composition of edges that are above the 95th percentile

Finally, we analyzed the composition of transit modes for the most critical edges/segments (Figure 9). Again, we limited the edges above 95th percentile of PWBC for each period and equity-seeking group combination. The aim was to find the transit modes that constituted the most important edges for each equity-seeking group. The early morning period was again excluded since the subway was not in operation over part of the period.

We found equity-seeking groups had a greater proportion of frequent bus edges for all times of day, except for carless households. Of the equity groups, black transit users consistently had the highest proportion for frequent buses. For bus routes, visible minorities and recent immigrants had higher proportions than the general populations for the morning peak and afternoon peak periods, but other equity groups had lower proportions than the general population. For the mid-day and evening periods, these edges make up a higher proportion for all equity groups. This would indicate that bus routes are more important for these equity-seeking groups, and they are more vulnerable to disruptions affecting frequent and non-frequent bus routes.

The general population and carless households have a higher proportion of streetcar edges. This may indicate that both groups are more likely to live or work near the downtown core than other equity group. Disruptions affecting the streetcar network would have a disproportionate impact on the general population and carless households. The high proportion for users who have no knowledge of English or French in the evening period may be because the downtown neighbourhoods have a higher proportion of population who have no knowledge of English compared to the Toronto average. They may also be more likely to have non-standard work hours as well.

The subway network does not have any major differences between equity-seeking groups and the general population, other than a slightly higher proportion for carless households. Carless households may be more likely to live immediately around subway stations due to their lack of vehicles in order to maintain their mobility, despite the higher property values associated with properties around the subway network (41).

CONCLUSION

Discussion

The results generally show the value of utilizing a time-expanded graph for vulnerability analysis. The coefficient of variation results shows that while it may be reasonable to use a static graph to represent a metro network, this is not appropriate for networks where a large proportion of ridership is served by surface transit in shared right of way. Low frequencies can significantly affect the shortest path from one departure time to the next which may make it difficult to determine if a node or edge is critical to the network, or if the network is vulnerable if the node or edge is removed.

Most transit agencies rely on bus or streetcar services to feed their rail networks. Even among rail transit systems, certain rail systems in North America have low frequencies, so a time-expanded approach should be considered as a method for transit network modelling in applications of resilience analysis. Our study shows the value of a time-expanded approach that accurately considers the interactions between low frequency routes and other routes. Beyond vulnerability studies, this approach for graph networks can be used for other types of graph analysis beyond vulnerability studies.

We also verified the impact multiple departure times would have on shortest paths and accessibility analysis. While we can reasonably assume that the travel time from an origin to a destination in a metro network would not have much variability, this is not true for bus or streetcar routes. Even for frequent transit routes, we observed much variation in the results.

In addition, equity-seeking users have different important nodes and edges compared to the general population, and their distribution within the network is different. They are more likely to be concentrated along frequent bus routes and are vulnerable to disruption involving those routes. In the Toronto context, snowstorms are a common cause of bus disruptions, so equity-seeking users are more likely to have inconsistent service during snowstorms.

With previous vulnerability studies focusing on metro networks and using a non-time-expanded approach, they may not capture comprehensively effects on different groups since vulnerabilities in the metro network may not be as important to equity-seeking users compared to the general population.

Finally, we can see that carless households have different patterns than other equity-seeking groups and exhibit patterns much like the general population. This shows the need for equity analysis to consider each equity group differently rather than as an aggregate.

Policy Context

For an equitable approach to improving system resiliency, system enhancements should target vulnerable and equity-seeking groups to increase the number of opportunities they need to improve their quality of life. This would mean cities should prioritize projects designed to improve bus system resilience, instead of the subway and downtown streetcar network. As the analysis shows that equity-seeking groups have more critical transit edges/stations and a higher entropy, re-prioritization would only bring the system to equality. To achieve equity, cities should focus on improving the quality of its bus system by making the routes less vulnerable to disruption, adding service on alternative routes, and adding transit priority on alternative routes to make them more attractive for pathfinding, much in the same way as LA Metro, which focused on improvements to the bus network over light rail after a settlement with the LA bus riders union (47).

In addition, research has shown that equity-seeking riders are the riders most likely to continue using transit through the COVID-19 pandemic, as they are unable to have other options, such as ready access to a vehicle, or work from home options (1,48). As equity-seeking riders will become the bulk of transit ridership in the immediate future after the pandemic, it becomes important to focus service changes on ways that will serve equity-seeking groups.

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AUTHOR CONTRIBUTIONS

The authors confirm contribution to the paper as follows: study conception and design: R. Liu, A. Shalaby; data collection: R. Liu; analysis and interpretation of results: R. Liu, A. Shalaby; draft manuscript preparation: R. Liu. All authors reviewed the results and approved the final version of the manuscript.

REFERENCES

1. Transit. Who's left riding public transit? A COVID data deep-dive. [Internet]. Transit App Blog. 2020. Available from: https://archive.transitapp.com/whos-left-riding-public-transit-hint-it-s-not-white-people-d43695b3974a

2. Dixit M, Chowdhury S, Cats O, Brands T, van Oort N, Hoogendoorn S. Examining circuity of urban transit networks from an equity perspective. Journal of Transport Geography. 2021 Feb;91:102980.

3. Sharma I, Mishra S, Golias MM, Welch TF, Cherry CR. Equity of transit connectivity in Tennessee cities. Journal of Transport Geography. 2020 Jun;86:102750.

4. Welch TF, Mishra S. A measure of equity for public transit connectivity. Journal of Transport Geography. 2013 Dec;33:29–41.

5. Mishra S, Welch TF, Jha MK. Performance indicators for public transit connectivity in multi-modal transportation networks. Transportation Research Part A: Policy and Practice. 2012 Aug;46(7):1066–85.

6. Palm M, Shalaby A, Farber S. Social Equity and Bus On-Time Performance in Canada's Largest City. Transportation Research Record. 2020 Nov;2674(11):329–42.

7. Liu R, Palm M, Shalaby A, Farber S. A social equity lens on bus bridging and ride-hailing responses to unplanned subway disruptions. Journal of Transport Geography. 2020 Oct;88:102870.

8. Idris AO, Habib KMN, Shalaby A. Dissecting the Role of Transit Service Attributes in Attracting Commuters: Lessons from a Comprehensive revealed Preference–Stated Preference Study on Commuting Mode-Switching Behavior in toronto, Ontario, Canada. Transportation Research Record. 2014 Jan;2415(1):107–17.

9. Taylor MAP. Public Transport Networks. In: Vulnerability Analysis for Transportation Networks [Internet]. Elsevier; 2017 [cited 2021 Jul 13]. p. 175–204. Available from: https://linkinghub.elsevier.com/retrieve/pii/B978012811010200006X

10. Derrible S, Kennedy C. Applications of Graph Theory and Network Science to Transit Network Design. Transport Reviews. 2011 Jul;31(4):495–519.

11. American Public Transportation Association. Public Transportation Ridership Report Fourth Quarter 2016 [Internet]. Washington DC: American Public Transportation Association; 2017 Mar. (Ridership Report). Available from: https://www.apta.com/wp-content/uploads/Resources/resources/statistics/Documents/Ridership/2016-q4-ridership-APTA.pdf

12. Litman T. Evaluating transportation equity. Victoria: Victoria Transport Policy Institute; 1999.

13. Allen J, Farber S. Sizing up transport poverty: A national scale accounting of low-income households suffering from inaccessibility in Canada, and what to do about it. Transport Policy. 2019 Feb;74:214–23.

14. Brick A. Incorporating and measuring social equity in transit service allocation [Masters]. [Seattle]: University of Washington; 2015.

15. Delbosc A, Currie G. Using Lorenz curves to assess public transport equity. Journal of Transport Geography. 2011 Nov;19(6):1252–9.

16. Karner A. Assessing public transit service equity using route-level accessibility measures and public data. Journal of Transport Geography. 2018 Feb;67:24–32.

17. Foth N, Manaugh K, El-Geneidy AM. Towards equitable transit: examining transit accessibility and social need in Toronto, Canada, 1996–2006. Journal of Transport Geography. 2013 May;29:1–10.

18. El-Geneidy A, Levinson D, Diab E, Boisjoly G, Verbich D, Loong C. The cost of equity: Assessing transit accessibility and social disparity using total travel cost. Transportation Research Part A: Policy and Practice. 2016 Sep;91:302–16.

19. Verbich D, El-Geneidy A. Public transit fare structure and social vulnerability in Montreal, Canada. Transportation Research Part A: Policy and Practice. 2017 Feb;96:43–53.

20. Dumas RA. Analyzing transit equity using automatically collected data [Thesis: M.C.P.]. [Cambridge, MA]: Massachusetts Institute of Technology; 2015.

21. Kramer A, Goldstein A. Meeting the public's need for transit options: characteristics of socially equitable transit networks. Institute of Transportation Engineers ITE Journal. 2015;85(9):23.

22. Lomax T, Margiotta R. Selecting travel reliability measures. Citeseer; 2003.

23. Mattsson L-G, Jenelius E. Vulnerability and resilience of transport systems – A discussion of recent research. Transportation Research Part A: Policy and Practice. 2015 Nov;81:16–34.

24. Berdica K. An introduction to road vulnerability: what has been done, is done and should be done. Transport Policy. 2002 Apr;9(2):117–27.

25. Jenelius E, Mattsson L-G. Road network vulnerability analysis of area-covering disruptions: A grid-based approach with case study. Transportation Research Part A: Policy and Practice. 2012 Jun;46(5):746–60.

26. King D, Aboudina A, Shalaby A. Evaluating transit network resilience through graph theory and demand-elastic measures: Case study of the Toronto transit system. Journal of Transportation Safety & Security. 2020 Aug 8;12(7):924–44.

27. Hijner AM. Indicators of spatial passenger delay propagation and their relation to topological indicators: A case study of the Washington DC metro network [Masters]. [Delft]: TU Delft; 2019.

28. Luo D, Cats O, van Lint H. Can passenger flow distribution be estimated solely based on network properties in public transport systems? Transportation. 2020 Dec;47(6):2757–76.

29. von Ferber C, Holovatch T, Holovatch Yu, Palchykov V. Public transport networks: empirical analysis and modeling. Eur Phys J B. 2009 Mar;68(2):261–75.

30. Jacob Rommel. Topological Analysis of the Evolution of Public Transport Networks [Masters]. [Stockholm]: KTH Royal Institute of Technology; 2014.

31. Bryan Whited. Application of Graph-Theoretic Indicators in Multi-Modal Public Transportation Networks [Masters]. [Edmonton]: University of Alberta; 2018.

32. Barthélemy M. Spatial networks. Physics Reports. 2011 Feb;499(1–3):1–101.

33. Freeman LC. A Set of Measures of Centrality Based on Betweenness. Sociometry. 1977 Mar;40(1):35.

34. Hong J, Tamakloe R, Lee S, Park D. Exploring the Topological Characteristics of Complex Public Transportation Networks: Focus on Variations in Both Single and Integrated Systems in the Seoul Metropolitan Area. Sustainability. 2019 Sep 29;11(19):5404.

35. Batagelj V. Understanding large temporal networks and spatial networks: exploration, pattern searching, visualization and network evolution. Southern Gate, Chichester, West Sussex: Wiley; 2014. 450 p. (Wiley series in computational and quantitative social science).

36. Quintero- Cano L. Graph theory based transit indicators applied to ridership and safety models. 2011 [cited 2021 Jul 13]; Available from: https://doi.library.ubc.ca/10.14288/1.0050719

37. Maduako I, Cavalheri E, Wachowicz M. Exploring the use of time-varying graphs for modelling transit networks. arXiv:180307610 [physics] [Internet]. 2018 Feb 16 [cited 2021 Jul 13]; Available from: http://arxiv.org/abs/1803.07610

38. CIRRELT/Polytechnique Montréal, Fortin P, Morency C, CIRRELT/Polytechnique Montréal, Trépanier M, CIRRELT/Polytechnique Montréal. Innovative GTFS Data Application for Transit Network Analysis Using a Graph-Oriented Method. JPT. 2016 Dec;19(4):18–37.

39. Shanmukhappa T, Ho IW-H, Tse CK. Spatial analysis of bus transport networks using network theory. Physica A: Statistical Mechanics and its Applications. 2018 Jul;502:295–314.

40. Zhang H. Structural Analysis of Bus Networks Using Indicators of Graph Theory and Complex Network Theory. TOCIEJ. 2017 Jan 30;11(1):92–100.

41. Hulchanski JD, University of Toronto, Cities Centre. The three cities within Toronto: income polarization among Toronto's neighbourhoods, 1970-2005 [Internet]. Toronto, Ont.: Cities Centre, University of Toronto; 2011 [cited 2020 Jan 26]. Available from: http://ra.ocls.ca/ra/login.aspx?inst=centennial&url=https://www.deslibris.ca/ID/226176

42. Rose A. 2016 TTS: Design & Conduct of the Survey. Toronto: University of Toronto Data Management Group; 2018. (Transportation Tomorrow Survey).

43. ICF Hannah Twaddell, ICF Beth Zgoda, Transit Cooperative Research Program, Transportation Research Board, National Academies of Sciences, Engineering, and Medicine. Equity Analysis in Regional Transportation Planning Processes, Volume 1: Guide [Internet]. Washington, D.C.: Transportation Research Board; 2020 [cited 2021 Jul 13]. Available from: https://www.nap.edu/catalog/25860

44. Karner A, Golub A. Comparison of Two Common Approaches to Public Transit Service Equity Evaluation. Transportation Research Record. 2015 Jan;2531(1):170–9.

45. Google. Create your data feed - Transfers [Internet]. Google; Available from: https://support.google.com/transitpartners/answer/6377424?hl=en

46. Stępniak M, Pritchard JP, Geurs KT, Goliszek S. The impact of temporal resolution on public transport accessibility measurement: Review and case study in Poland. Journal of Transport Geography. 2019 Feb;75:8–24.

47. Grengs J. Community-Based Planning as a Source of Political Change: *The Transit Equity Movement of Los Angeles' Bus Riders Union*. Journal of the American Planning Association. 2002 Jun 30;68(2):165–78.

48. Shamshiripour A, Rahimi E, Shabanpour R, Mohammadian A (Kouros). How is COVID-19 reshaping activity-travel behavior? Evidence from a comprehensive survey in Chicago. Transportation Research Interdisciplinary Perspectives. 2020 Sep;7:100216.